5A Verification of P-U intensity calculation

5A.1 Summary

This chapter is based on [6].

For several decades P-P probes have been used to measure sound intensity. In the last few years P-U probes have also become available. One of the advantages of these small P-U intensity probes is that they can be used in environments with high p/I indexes. Because the sound pressure and particle velocity are measured in one spot, there is no spacing (and thus bandwidth) problem, as is the case with P-P probes. The calculation of the intensity with the P-U method is fairly easy, but the characterization and implementation of the calibration values can sometimes be confusing. The accuracy of the calibration is especially important in acoustic fields with high reactivity. In this article a broadband calibration technique based on application of a spherical source is discussed. A new procedure is presented to check if the sensor responses are properly corrected. The results are compared to P-P probes, but also to a regular pressure microphone in a defined sound field (of a monopole source). Some real life structures have been used to verify intensity measurements through P-U probes.

5A.2 Introduction

Sound intensity is a useful tool for localizing, quantifying and ranking sound sources. Intensity is the time averaged product of sound pressure and particle velocity. Since many years, it has been measured with so-called P-P probes where the particle velocity is derived from the pressure difference between two microphones positioned at a certain distance with respect to each other. Sound pressure on the other hand, is obtained by averaging the signals from two microphones.

In 1994 a sensor is invented that measures the particle velocity directly. This sensor, together with a microphone, can measure the intensity in one spot, whence avoiding the spacing problems encountered in P-P probes. This sensor can be used in environments, with high levels of background noise or reflections, where it is not possible to use P-P probes (high p/I index).

In this paper the calibration procedure for P-U probes is described. To check the calibration parameters and to validate the intensity measurement of the P-U probe, the results are then compared not only with a P-P probe but also with a normal pressure microphone, knowing that the radiated sound intensity level (in dB) generated by a monopole sound source placed in an anechoic room, equals the sound pressure level (in dB). In such a special situation one can also determine the sound intensity using a single pressure microphone. Although the sound source used in this investigation is slightly different from a point source, the deviation from a perfect monopole can be properly modeled.

When discrepancies between P-P and P-U intensity probes are observed, it is not always easy to tell which method would be the more accurate one. With the pressure microphone as an extra reference, one possesses an additional tool to discern the true intensity value.

Here, the parameters like coherence and the relation between active and reactive intensity are used to investigate the error associated with both P-U and PP probes.

5A.3 Intensity calculation

P-P intensity method

The P-P measurement procedure makes use of two microphones. The sound pressure is the average of the two corresponding pressure signals. The intensity is calculated at the center of the space separating the two microphones. The P-P intensity is then obtained by the following relation [1]:

$$\hat{I}_{pp} = \langle \hat{p}\hat{u} \rangle_t = \left\langle \frac{p_1(t) + p_2(t)}{2} \int_{-\infty}^{t} \frac{p_1(\tau) - p_2(\tau)}{\rho \Delta r} d\tau \right\rangle_t \tag{5A.1}$$

Where the sign ˆ indicates an estimated quantity and $\langle \ \rangle_t$ averaging over time, ρ is the density of air and Δr is the microphone separation.

Errors associated with P-P probes

Application of finite difference approximation, scattering and diffraction, microphone size and instrumentation phase mismatch can all be sources of errors, when implementing PP intensity technique. The accuracy of the finite-difference approximation depends on the microphone separation. For a plane wave of axial incidence, the finite difference error is:

$$\hat{I}_{pp} / I = \frac{\sin k\Delta r}{k\Delta r} \tag{5A.2}$$

Where $k = 2\pi f / c$ is the wave number and I is the "true" intensity (i.e. unaffected by phase mismatch). Unless the measurement is compensated for phase mismatch, the PP probe microphones have to be phase matched quite accurately. The state-of-the-art PP probe microphones are matched to a maximum phase difference of 0.05° below 250Hz. Above this frequency, the phase difference is proportional to the frequency (typically about 0.2° at 1kHz). A small phase mismatch error φpe leads to a bias error approximated by [1], [3]:

$$\hat{I}_{pp} \cong I_{pp} - \frac{\varphi_{pe}}{k\Delta r}\frac{p_{rms}^2}{\rho c} = I\left(1 - \frac{\varphi_{pe}}{k\Delta r}\frac{p_{rms}^2 / \rho c}{I}\right) \tag{5A.3}$$

Where I is the "true" intensity (i.e. unaffected by phase mismatch), p_{rms} is the rms value of the sound pressure, ρ is the density of the air and c is the speed of sound. This expression shows that the effect of a given phase error is inversely proportional (to the frequency and) the microphone separation distance and proportional to the ratio between mean square sound pressure and sound intensity. If this ratio is large then even the small phase errors mentioned above, will generate significant bias errors. Because of the phase mismatch, it will rarely be possible to make reliable measurements below ~80Hz unless a longer spacer than the usual 12 or 25 mm is used.

The ratio of the phase error to the product of the frequency and the microphone separation i.e. $\varphi_{pe} / k\Delta r$, is usually measured in the form of the so-called "pressure-residual intensity index, or p/I index", by exposing the two pressure microphones to the same pressure in a small coupler. Modern sound intensity analyzers automatically determine the ratio of the mean square pressure to the intensity, $\left(p_{rms}^2 / \rho c\right)/I$, during the intensity measurements. This so-called pressure-intensity index is an indication of the bias error discussed above.

Active and reactive intensity in P-U probes

A P-U sound intensity measurement system uses a fundamentally different approach by directly measuring the particle velocity. The sound intensity is simply the time average of the instantaneous product of the pressure and particle velocity signal [1], [3]:

$$\hat{I}_{pp} \cong I_{pp} - \frac{\varphi_{pe}}{k\Delta r}\frac{p_{rms}^2}{\rho c} = I\left(1 - \frac{\varphi_{pe}}{k\Delta r}\frac{p_{rms}^2 / \rho c}{I}\right) \tag{5A.4}$$

Where the latter expression is based on the complex representation of harmonic variables. This approach bypasses therefore, the errors inherent to the finite difference approximations.

Whereas the (active) intensity describes the net flow of sound energy, the reactive intensity describes the non-propagating part of the energy that is merely flowing back and forth, corresponding to the instantaneous particle velocity being in quadrature with the sound pressure. Many sources are strongly reactive at low frequencies (or/and at near field) where they mainly generate evanescent waves. At the acoustic near field, the air is essentially moving back and forth as if it were incompressible. The reactive intensity is expressed by:

$$J_{pu} = \tfrac{1}{2}\mathrm{Im}\{pu\} \tag{5A.5}$$

Errors associated with P-U probes

The error in P-U probes is not affected by the p/I index, but depends mainly on the reactivity of the sound field. If the reactivity (the ratio of the reactive to the active intensity in logarithmic scale) takes a large value, as for example in the near field of a source, then even a very small phase mismatch error between the two transducers may lead to a considerable bias error, as can be seen from the expression:

$$\hat{I}_{pu} = \mathrm{Re}\left\{S_{pu} e^{j\varphi_e}\right\} = I\cos\varphi_e - J\sin\varphi_e = I\left(1-\varphi_e \frac{J}{I}\right) = I\left(1-\varphi_e \tan\varphi_{field}\right) \qquad (5A.6)$$

Where $\hat{I}_{pu}$ is the measured intensity, S_{pu} is the 'true' cross spectrum between the sound pressure and the particle velocity, φ_e is a small phase error between the measured and the 'true' particle velocity, I is the 'true' intensity, and J is the 'true' reactive intensity. The phase φ_{field} is the phase shift of the sound field.

Relation (6) shows that substantial P-U phase errors can be tolerated if J<<I. Under such conditions, even a phase mismatch of 35°, for example, would result in a bias error of less than 1dB. In other words, the phase calibration is critical when the measurements are carried out in presence of strong reactive conditions (i.e. near field), but not at all critical for far field measurements. The "reactivity" (the ratio of the reactive to the active intensity) is therefore an indication to whether this source of error is of concern or not [1], [3].

5A.4 P-U Probe calibration

Presently, a broad band (20Hz-20kHz) calibration technique will be described that makes use of a spherical sound source with a known acoustic impedance [4], [5]. A P-U probe and a reference microphone are placed at a certain distance from this source. The particle velocity is calculated from the known source impedance, while the sound pressure is measured by a reference microphone. To calibrate the particle velocity at low frequencies, the sound pressure is measured inside the sphere. This pressure is proportional to the movement of the loudspeaker membrane from which the particle velocity in front of the loudspeaker can be derived.

High frequency calibration

The reference microphone and the pressure-velocity probe are positioned at nearly the same position in front of the spherical source. Because the pressure field is similar for both probes the pressure sensor can directly be compared to the reference microphone. Given the sound pressure measured by the reference microphone and the known impedance at the measurement position, both the amplitude and phase response of the velocity sensor can be characterized.

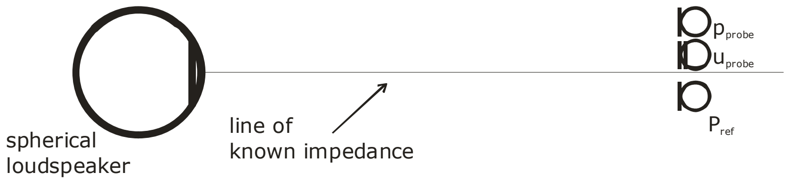


Fig. 5A.1: The piston in a sphere set up

The spherical source consists of a rigid sphere in which a small loudspeaker is placed. Such a source can be modeled as a sphere with radius *a* and a moving piston with radius *b*. The relation between sound pressure and particle velocity (the acoustic impedance) on the axis of the piston is given by [4]:

$$\frac{u(r)}{p(r)}=\frac{1}{Z_{pu}(r)}=\frac{j}{\rho c}\frac{\sum_{m=0}^{\infty}\left(P_{m-1}(\cos\alpha)-P_{m+1}(\cos\alpha)\right)\frac{h_m^{'}(kr)}{h_m^{'}(ka)}}{\sum_{m=0}^{\infty}\left(P_{m-1}(\cos\alpha)-P_{m+1}(\cos\alpha)\right)\frac{h_m(kr)}{h_m^{'}(ka)}} \tag{5A.7}$$

Where *r* is the distance form the centre of the sphere, α =arcsin(*b/a*). P_m is the Legendre function of the order *m*, h_m is the spherical Hankel function of the second kind and order *m*, and h'_m is its derivative. Although the impedance on the axis of the spherical source is described by the complex expression (7), the resulting impedance is quite similar to the acoustic impedance of a monopole source, given by the expression:

$$Z(r)=\rho c\frac{ikr}{1+ikr} \tag{5A.8}$$

The measured impedance can seem rather noisy because of room reflections. This effect is reduced by smoothing the results in the frequency domain through a moving average filter.

Low frequency calibration

The PU probe sound pressure element can be calibrated in the 20Hz-10kHz range, because its calibration is based on the comparison with the output of the adjacent reference microphone. Both sensors are omni-directional and it doesn't matter if the output is caused by background noise or by the spherical source. However, at low frequencies (below ~100-200Hz) the noise generated by the source does not sufficiently exceed the background noise and eq. (7) can not be applied anymore. To calibrate at low frequencies in an ordinary room, the reference microphone must be placed inside and the velocity sensor, close to the sphere.

There is a simple relation, between the interior pressure and the particle velocity at the probe position, for low frequencies below the first internal acoustic resonance of the sphere. Due to the continuity condition, the particle velocity in front of and very close to the membrane is almost equal to the velocity on the surface of the membrane itself. The relation between the sound pressure in the sphere and the particle velocity in front of the sphere is given by:

$$u_{piston}=-\frac{i\omega V_0}{\gamma A_0 p_0}p_{ref} \tag{5A.9}$$

Where ω is the angular frequency, V_0 is the interior volume of the sphere, A_0 the surface area of the moving piston, p_0 the ambient pressure and γ is the ratio of specific heat of gas at a constant pressure to a constant volume (1.4 for normal air). An adiabatic compression and rarefaction of the air in the sphere is assumed. The relation between the particle velocity in front of the piston, which equals the normal velocity on its surface (u_n), and the particle velocity at a distance *r* from the centre of the sphere is given by expression (10).

$$u(r) = -\frac{u_n}{2}\sum_{m=0}^{\infty}\left(P_{m-1}(\cos\alpha) - P_{m+1}(\cos\alpha)\right)\frac{h'_m(kr)}{h'_m(ka)} \tag{4A.10}$$

Combining the equations yields a relation between the particle velocity at position *r* and the pressure in the source. When the velocity sensor is positioned in the very near field of the source this can be approximated by a straight line. At approximately 1300Hz there is a dip in the amplitude response of the transfer function u_n / p_{ref} of the low frequency calibration. This is due to the first internal acoustic mode inside the sound source and can be approximated by a simple cosine function. Because parameters V_0, A_0 and *r* are not known exactly cannot be solved in absolute sense. The phase of the frequency response is precisely known because it does not depend on these parameters. However, by not knowing the exact scalar values only an amplitude error is made which is constant over the frequency range.

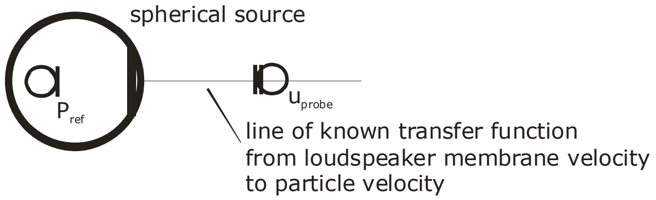


Fig. 5A.2: Low frequency calibration set up. The pressure microphone is put inside the sphere.

Combining the low and high frequency approach

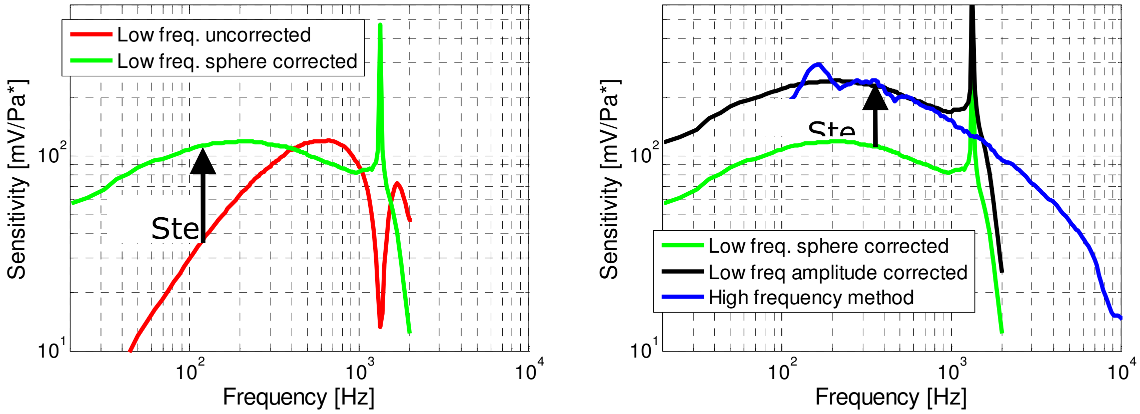


Fig. 5A.3: Correction of the velocity amplitude of the low frequency calibration.

The low frequency response (Fig. 5A.3, red line, step 1) is now corrected as is mentioned above. In the corrected response (green line) there is a peak around the frequency of the dip. Here the cosine approximation gives extreme values and values close to this frequency are therefore not used. Because the absolute value of the sensitivity amplitude is known from the high frequency calibration, it can be linked to the low frequency calibration curve. A different constant value (for the constants V_0, A_0) is used and the low frequency calibration curve (Fig. 5A.3, green line, step 2) is shifted so the high frequency curve overlaps in the medium frequency range (~200-800Hz).

The sensitivity of particle velocity sensors is frequency dependent as can be clearly observed in Fig. 5A.3. For practical applications it is convenient to use an analytical model and fit the result to the measured calibration curve [5]. Also one must note that the pressure microphones in P-U probes have slightly frequency dependent sensitivities that must also be estimated through a model.

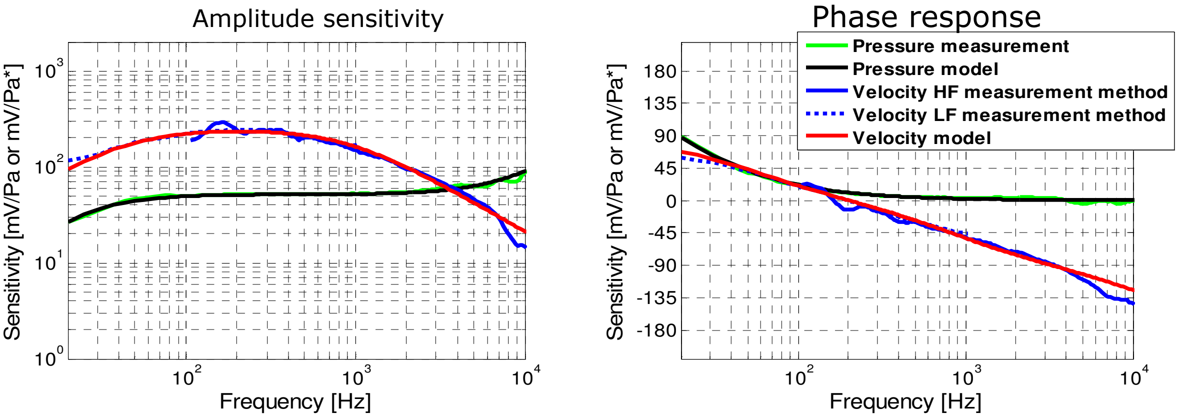


Fig. 5A.4: Example of the broad band calibration response of pressure and velocity (1 Pa* is the particle velocity corresponding to a 1 Pa plane pressure wave : $1\,\mathrm{Pa}^* = 1\,\mathrm{Pa}/\rho c$).

5A.5 Sound pressure versus sound intensity

Sound pressure and particle velocity are linked through the acoustic impedance which is the ratio between the sound pressure and particle velocity. The sound intensity is the product of sound pressure and the particle velocity. It is therefore understandable that sound intensity and sound pressure be also related through the acoustic impedance. The relation between the (active) sound intensity and the acoustic impedance is given by:

$$\frac{I(r)}{p^2(r)/\rho c} = \frac{\mathrm{Re}\{p(r)\cdot u(r)\}}{p^2(r)/\rho c} = \frac{\mathrm{Re}\{p^2(r)/Z(r)\}}{p^2(r)/\rho c} = \mathrm{Re}\left\{\frac{\rho c}{Z}\right\} \qquad (5A.11)$$

One specific case is point source, where the real part of the impedance is equal to unity:

$$Z^{-1}_{point\,source}(r) = \frac{1}{\rho c}\left(1+\frac{1}{ikr}\right) \qquad (5A.12)$$

Whence:

$$\frac{I(r)}{p^2(r)/\rho c} = \mathrm{Re}\left\{\frac{\rho c}{Z}\right\} = 1 \tag{5A.13}$$

Due to different reference values (SPL re. 20μPA, SIL re. 1nW/m2), the sound pressure expressed in dB is equal, in the present case, to the intensity in dB.

It is difficult to construct a point source capable of generating sufficiently high sound levels in a broad frequency range. For this reason a "piston on a sphere" source is used instead. This is the same sound source as was used during calibration process. In this case the specific acoustic admittance is described by eq. (A5.7).

5A.6 Pressure, P-P and P-U measurements

The P-P and P-U measured intensity can directly be compared to the sound pressure because the radiation impedance of the spherical source is known. Three spacings (50mm, 25mm and 12mm) are used for the P-P probe in order to compare the results in the whole frequency range. A strong reactive field is present at small measurement distances and at low frequencies, one of the most complex situations for the P-U technique. The P-P probe should on the other hand, have a relatively small error (low p/I index), because the background noise level is quite low. Here, the source has been powered with a white noise signal. The following three measurement setups are used:

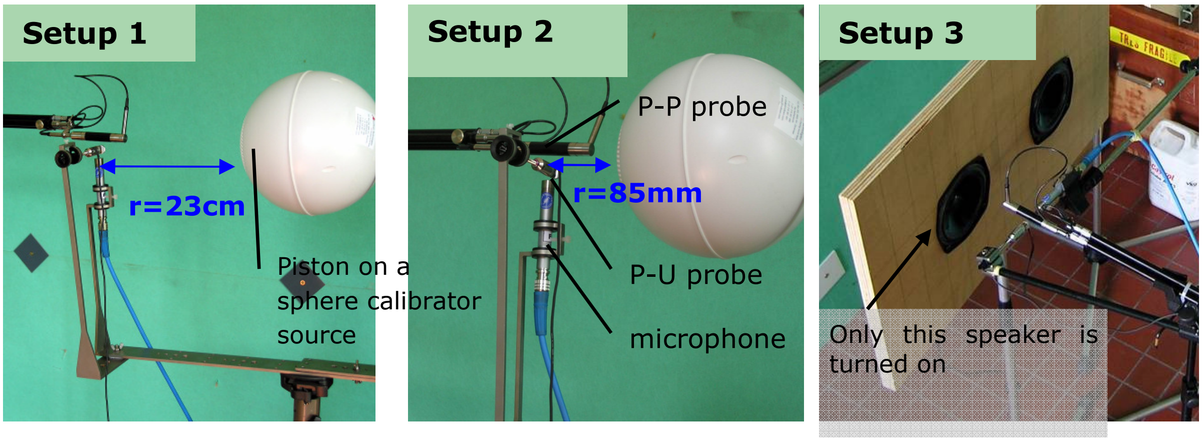
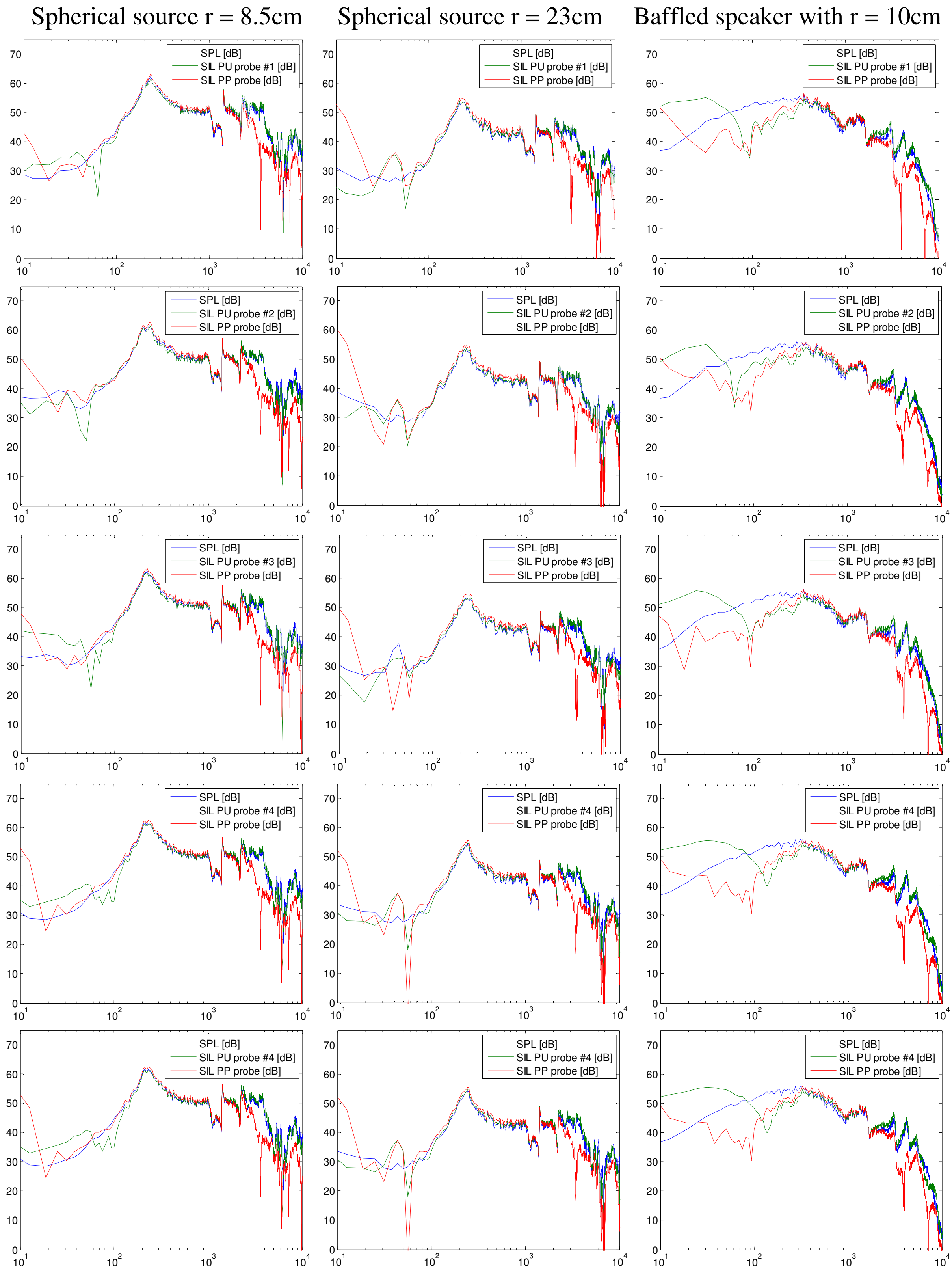


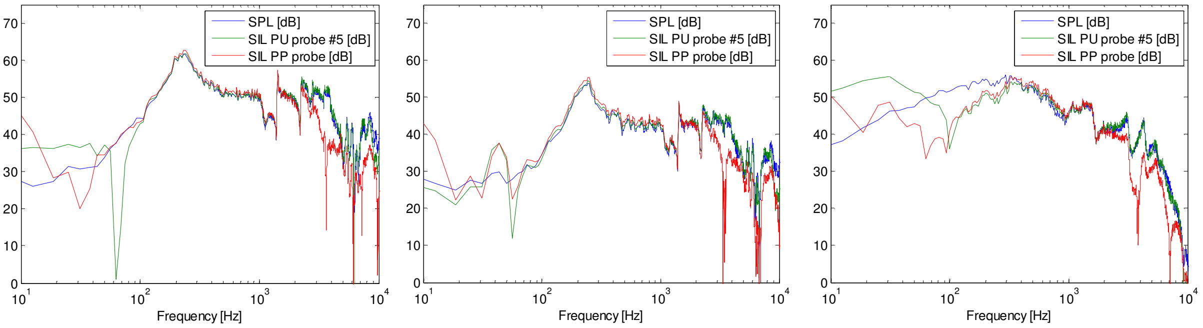
Fig. 5A.5: Left and middle: Piston on a sphere calibrator. Right: Baffled loudspeakers.

Measurements of sound pressure and intensity (P-P with 50mm spacer and P-U)

Measurements with 5 P-U probes are performed to study the variations among the probes. Below, the left hand figures represent the results corresponding to the probes positioned at 8.5cm from the spherical source, and the middle ones correspond to a 23cm distance. The right hand figures illustrate the results associated with one of the baffled speakers distanced by 10cm from the probes.

Verification of P-U intensity calculation





Difference between sound pressure and P-P (50mm spacer), P-U intensity

The sound source used in setup 1 and 2 is not an exact point source. The difference between the sound pressure and intensity calculated from eq. (7) is shown in red in Figure 6 and Figure 7. Also, measured differences between the sound pressure and the P-P and P-U intensities from the measurements above are shown. Measurement Ipp 1 to 5 corresponds here to the same P-P probe at different five measurement moments.

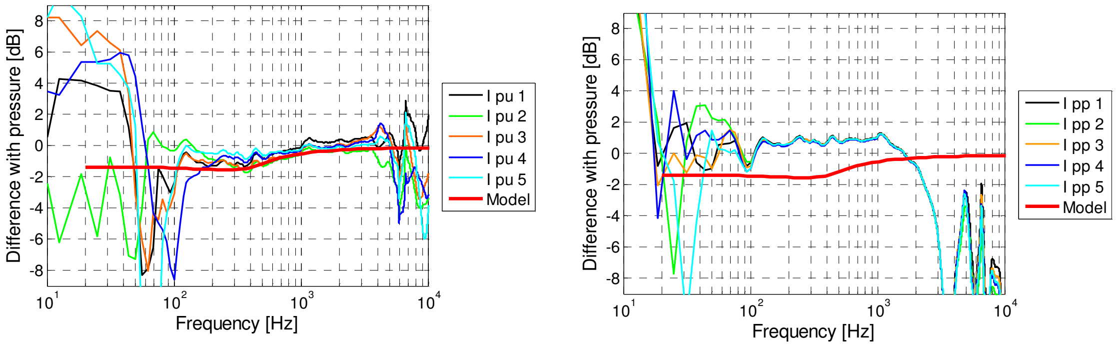


Fig. 5A.6: Difference between the pressure and intensity 85mm from the spherical source.

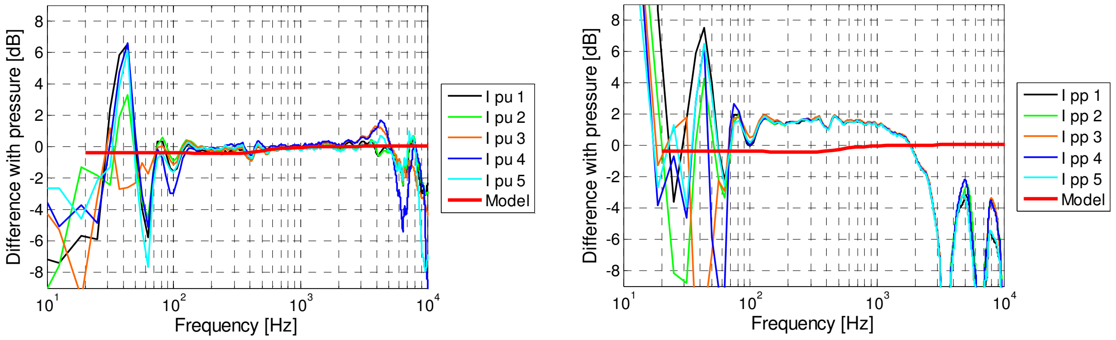


Fig. 5A.7: Difference between the pressure and intensity 23cm from the spherical source.

The P-U intensity is distorted at 85mm distance below ~150Hz because of the strong reactivity of the source. Further from the source (23cm) however, there seems to be a better agreement with the sound pressure. This means that the calibration values are quite accurate and that the raw outputs of the sensors are properly adjusted by the model. In this example the intensity measured by the PP probe is with ~2dB higher, the reason being unclear. As can be expected, the deviations are higher at above 1.5kHz because of the relatively large microphone spacing i.e. 50mm. The impedance of setup 3 (baffled speaker) is different from a point source. From the measurements it can be seen that the relation between pressure and intensity is not anymore valid at low frequencies. But the results seem similar to a point source at above 300Hz and the measurement can still be valid enough to roughly check the intensity calculation.

Pressure and intensity using different P-P spacers

The same measurement procedure is repeated, but now using a P-P probe with 25mm 12mm spacers. As expected, the difference with the P-U intensity becomes smaller at higher frequencies:

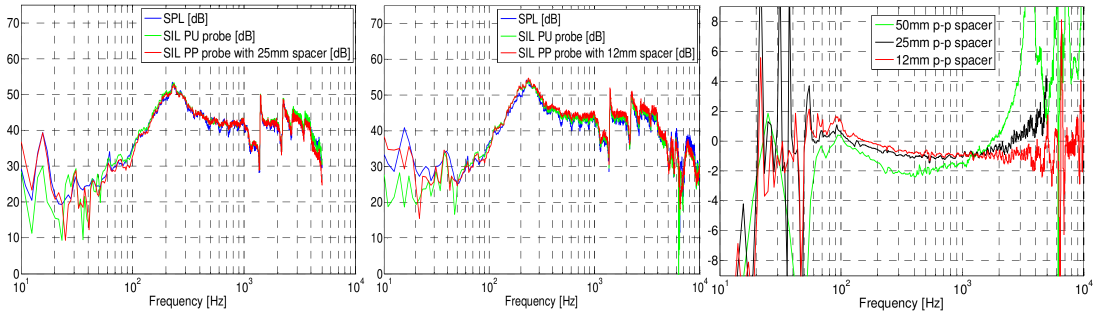


Fig. 5A.8: Sound pressure, P-P and P-U intensity. Left: with 25mm P-P spacer. Middle: with 12mm P-P spacer. Right P-P and P-U intensity difference with three P-P probe spacers.

Pressure comparison

In some of the measurements, the P-P probe intensity level is 1 to 2 dB higher than reference microphone pressure and P-U intensity levels. The first logical approach to this problem would be to verify if such a broad band difference is caused by inaccurate P-P probe sensitivity values. The average sound pressure of the P-P microphone pair should equal the sound pressure of the reference microphone. This is the case here, as can be seen below, where the average pressure matches the pressures measured by both reference microphone and the P-U probe and whence, overruling the sensitivity issue.

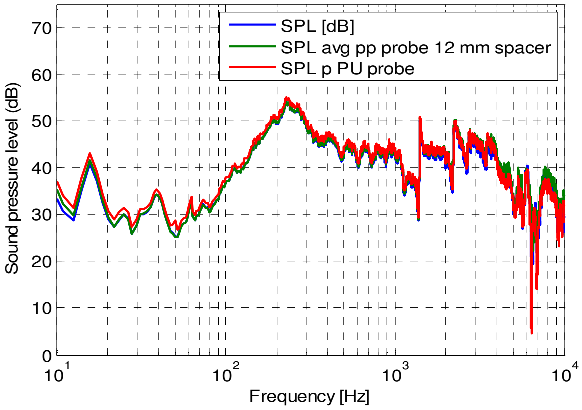


Fig. 5A.9: Average sound pressure of the P-P probe (12mm spacer) compared to the reference microphone and the P-U microphone.

P-P and P-U coherence

A sound pressure sensor (microphone) is more sensitive to the background noise than a velocity sensor for several reasons: In the near field the air is relatively incompressible and there is a relatively small level of sound pressure compared to particle velocity. Furthermore, an increased level of external (background) noise lowers the microphone signal-to-noise ratio. In the present case, the pressure rises when the background noise reflects back from the spherical source surface. In other words, a multidirectional noise causes a pressure buildup. Contrarily, such reflections lead to a drop in particle velocity. This is because the velocity sensor detects the velocity signal in only one direction due to its figure-of-eight sensitivity pattern.

For these reasons, the coherence is not a good indicator of the P-P probe error. It can be seen that the coherence between the P-P probe microphones is almost equal to one, down to 10Hz. However, the pressure is not dominated only by the source (at 23cm distance). The coherence between the speaker input signal and the outside pressure microphone drops sharply below 100Hz. This implies that below 100Hz the sound pressure is dominated mainly by the background noise rather than the sound source.

For similar reasons, the velocity at lower frequencies is more coherent to the source input signal. The PU probe pressure sensor is affected by the background noise, but since only the correlated part with the velocity sensor is taken, the influence is reduced. The PP intensity probes however use two pressure sensors and are therefore more vulnerable to the background noise, particularly at lower frequencies.

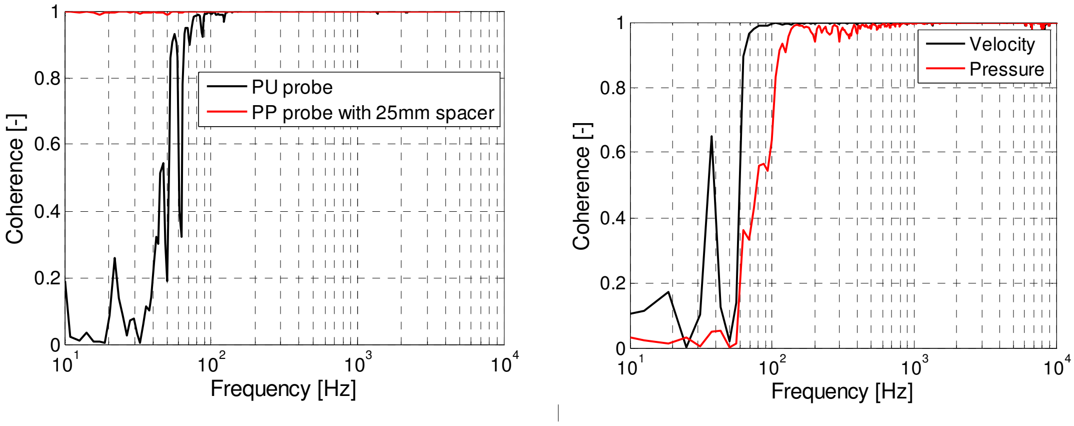


Fig. 5A.10: Left: Coherence between each probe's associated sensors. Right: Coherence of pressure and the velocity to the input signal of the sound source

P-U probes and field reactivity

Reactivity can be calculated. The measurements show a good agreement between the measured intensity, by the P-U probe, and calculated ratio of reactive and active intensity. As a rule of thumb for accurate P-U intensity measurements, the reactive intensity should not exceed the active intensity by more than ~5dB. [2]

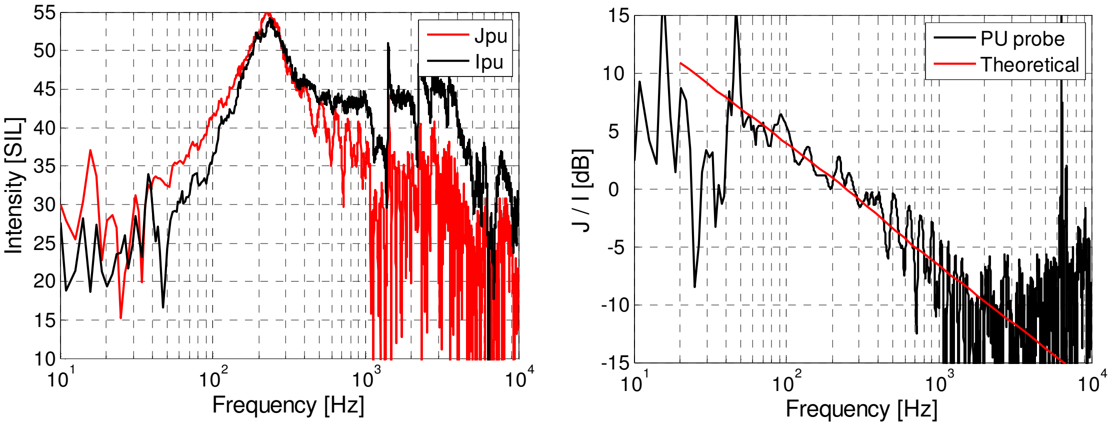


Fig. 5A.11: Left: Active and reactive P-U intensity. Right: Measured and theoretical J/I ratio.

5A.7 Conclusion

P-U and P-P probe intensity techniques are often compared, however, it is difficult to decide which method's results to trust, in case there are differences. In this paper the sound pressure is used as an additional criterion to be considered in a sound field with a known acoustic impedance. Pressure microphones are commonly available and can be used in the whole frequency range and do not suffer from limitations such as microphone spacing in P-P probes. This is a quick way to check the validity of the (PP and PU) intensity probes as well as the corresponding calibration values. In this study a piston on a sphere sound source is used as an alternative to a true point source. To compare the pressure to the velocity a correction is applied for the slightly different impedance of this piston on a sphere source.

In some examples provided here, the P-P intensity seems to be slightly overestimated, the reason for that being unclear. Even though the measurements were done in a quiet environment, below 100 Hz the sound pressure still seems to be dominated by the background noise. The particle velocity is less affected by the background noise and because only the correlated part between pressure and velocity is considered, the P-U intensity can be considered as a net improvement over that provided by P-P probes.

The error associated with P-U method on the other hand, grows as the field becomes more reactive, such as in the acoustic near field of a source.

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