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Comparison of inverse methods and particle velocity based techniques for transfer path analysis

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Direct sound field visualization is not always the best way to assess complex noise problems. Maps of sound pressure, particle velocity or sound intensity in the vicinity of a source might not be directly related to the pressure contribution for a given position. Transfer path analysis has been implemented for many years to evaluate this case scenario, which requires using information about the environment and the sound source. Inverse methods commonly require a large number of transfer function measurements along with special measurement conditions. On the other hand, particle velocity methods rely on measuring the reciprocal transfer path and the velocity distribution of the vibrating surfaces directly. This paper presents the theoretical bases of the two principles and compares the advantages and disadvantages of the two methods applied to real industrial applications.

1 Introduction

One of the most significant current discussions in industrial acoustics is focused on finding suitable measurement techniques for relating sound sources and noise levels at specific locations. There are two fundamental aspects that shall be solved separately. First of all estimating the sound pressure “contribution” from different radiating surfaces. Secondly predicting how such “contributions” could change when an acoustic treatment is applied. Most of measurement methods focused on solving these problems are techniques based on Transfer Path Analysis (TPA). Some of the methods localize the pressure contribution of airborne noise sources (Airborne TPA) whereas another techniques study a coupled problem regarding as well the forces which are exciting the panels via structural paths. Figure 1 presents a sketch of the problem addressed.

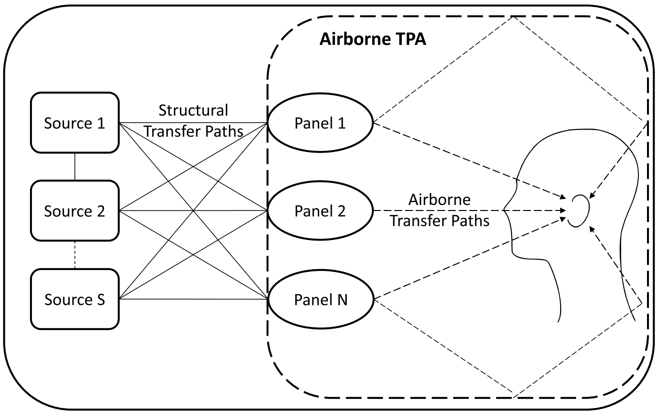

Figure 1: Sketch of a typical Transfer Path Analysis problem regarding structural and airborne noise sources.

Following the common ground between most widespread techniques, the complex radiating structure is usually discretized into multiple vibrating surface areas denoted as ‘panels’ . Then, their degree of “contribution” should be defined in order to rank which panels has a stronger influence on causing the sound pressure at the evaluated position. This problem is normally referred as “Panel Contribution Analysis”. In the technical literature, several experimental techniques can be found that assess this problem. Most commonly used methods within Airborne TPA are“windowing” techniques [1], substitution monopole techniques (SMT) [2, 3], matrix inversion methods [4], direct particle velocity measurements [5, 6, 7], beamforming [8, 9, 10] and holographic technologies using pressure arrays [11]. On the other hand, using one of the mentioned methods along with operational forces and structural transfer paths of the vibrating structure can be seen as a general TPA approach [12]. A general overview of the current techniques is presented in Figure 2. Either structural or airborne TPA have been divided into two main steps: excitation characterization (top) and medium evaluation (bottom). The most common measurement methods for performing each of the steps are presented in Figure 2 with italics.

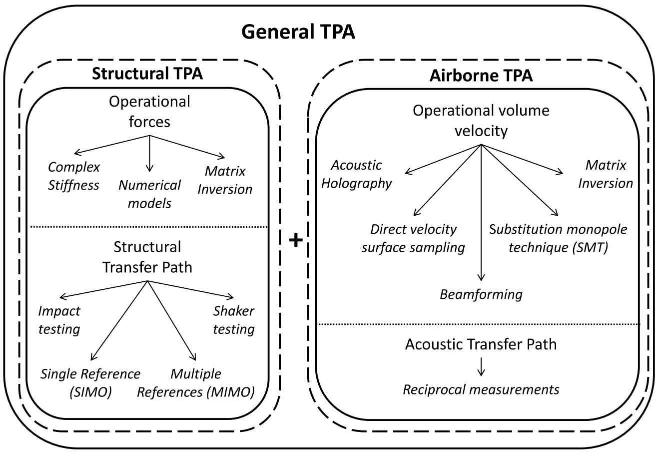

Figure 2: General overview of the principal transfer path analysis (TPA) techniques with their corresponding measurement procedures.

Nowadays there is an unambiguous relationship between novel velocity-based techniques and traditional pressure-based methods. The use of particle velocity sensors (or Microflowns) have been shown very promising for industrial applications over the years [5, 6]. However, the widespread use of pressure microphones have limited the development of alternative methodologies. This paper aims to clarify the advantages and disadvantages on using the velocity-based measurement techniques compared with conventional pressure-based methods which rely on matrix inversion. A theoretical basis is presented along with an experimental evaluation of both principles and a discussion focused on their corresponding limitations.

2 Theory: Airborne TPA

In order to assess the underlying theory behind pressure and velocity based methods for Airborne TPA, a general approach shall be taken. Let us start by defining a complex structure **S** which surface excites the sound field when it is under operating conditions. Then, an infinitesimal small area **M** can be defined for studying how different areas of the structure “contribute” to the position of **M**. Figure 1 shows a sketch of the scenario described.

The theoretical derivations of an expression for calculating the pressure contribution at **M** follow Hald [13] and Kinsler [14]. First of all, it is necessary to define two different measurement conditions: when a monopole source at **M**

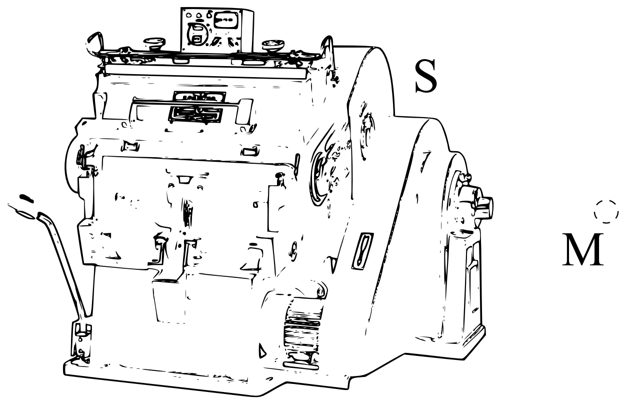

Figure 3: Structure and field point involved in the derivation.

is exciting the sound field (reciprocal transfer function measurements); and when the monopole is switched off and the structure **S** is producing the noise (noise measurements).

Two set of variable can be distinguished depending on the measurement conditions. p^{TF} and u^{TF} are defined as the pressure and particle velocity during the reciprocal transfer function measurements. On the other hand, p and u are the pressure and particle velocity during the noise measurements.

As have been pointed out by Hald, for deriving an expression which describes the fundamental basis of panel noise contribution analysis it is necessary to start using the definition of acoustic reciprocity [14],

$$\int_{\mathbf{M}} (p^{TF}u - pu^{TF})d\mathbf{M} + \int_{\mathbf{S}} (p^{TF}u - pu^{TF})d\mathbf{S} = 0 \qquad (1)$$

The integral of particle velocity u across the entire surface **M** will be zero due to there is no net energy going throughout M during the noise measurements. Furthermore, the pressure p can be integrated over **M** during the noise measurements obtaining the reference pressure p_r. Besides, integrating the particle velocity over **M** during the transfer function measurement will lead to obtain the volume velocity of the monopole source Q. Consequently, the previous expression leads to

$$-p_rQ + \int_{\mathbf{S}} (p^{TF}u - pu^{TF})d\mathbf{S} = 0 \qquad (2)$$

Then, the pressure at the reference position can be defined as

$$p_r = \int_{\mathbf{S}} \left(\frac{p^{TF}}{Q}u - p\frac{u^{TF}}{Q} \right) d\mathbf{S} \qquad (3)$$

Eq. (3) presents the fundamental equation for most of velocity-based panel noise contribution methods. It relates the pressure at the reference position p_r with the combination of particle velocity u and pressure p along with acoustic transfer functions p^{TF}/Q and u^{TF}/Q measured across the structure S.

2.1 Reference-Related method

Arbitrary signals have been considered on the derivation of a general expression for expressing the pressure at a given position (see Eq. (3)). Nonetheless, for real scenarios it would be necessary to deal with random signals [15]. Moreover, Eq. (3) can only be implemented directly if all the pressure and velocity distribution around **S** are acquired simultaneously. Otherwise, global phase differences between source velocities would be lost between different sessions. In order to overcome this problem a novel approach was first presented in [16]. The main idea is to exploit the potential of having a fixed reference sensor to synchronize multiple sessions without loosing phase information of the panels. Nevertheless, the reference-related technique is focused on solving the fundamental expression given in Eq. (3) using relative phase information, instead of preserving global phase terms. Hence, Eq. (3) is rewritten firstly multiplying by the complex conjugate version of the pressure reference p_r^* and then taking the expected values $E(...)$ of the different terms,

$$E(p_rp_r^*) = \frac{1}{A_m}\int_{\mathbf{S}} \left[E\left(\frac{p^{TF}}{u_m}\right)E(up_r^*) - E(pp_r^*)E\left(\frac{u^{TF}}{u_m}\right) \right] d\mathbf{S} \qquad (4)$$

where u_m is the particle velocity at **M** during the transfer path measurements; and A_m is the area of **M**.

Eq. (4) can be now expressed by a combination of auto spectras and cross-spectras, i.e.

$$S_{p_rp_r} = \frac{1}{A_m}\int_{\mathbf{S}} \left(\frac{S_{p^{TF}u_m}}{S_{u_mu_m}} S_{up_r} - S_{pp_r}\frac{S_{u^{TF}u_m}}{S_{u_mu_m}} \right) d\mathbf{S} \qquad (5)$$

where $S_{p_rp_r}$ is the autospectrum of the pressure reference; $S_{p^{TF}u_m}$ is the cross spectrum between the pressure at S and velocity at M both during the transfer function measurements; $S_{u_mu_m}$ is the autospectrum of u_m; S_{u^{TF},u_m} is the cross spectrum between the velocity at S and velocity at **M** both during the transfer function measurements; $S_{(}up_r)$ is the cross-spectrum between velocity at **S** and the reference pressure; and S_{pp_r} is the cross spectrum between the pressure at S and pressure at **M** both when the cavity is exciting the sound field.

In practical cases, the surface **S** has to be discretized by dividing it into a limited number of panels N. Consequently, Eq. (5) leads to

$$S_{p_rp_r} = \frac{1}{A_m}\sum_{n=1}^{N} \left(\frac{S_{p^{TF}u_m}}{S_{u_mu_m}} S_{up_r} - S_{pp_r}\frac{S_{u^{TF}u_m}}{S_{u_mu_m}} \right) A_n \qquad (6)$$

where A_n defines the area of each panel n.

This theoretical approach is the base of validated step-by-step measurement methods such as PNCAR (Panel Noise Contribution Analysis Reference-Related) [5], but also of scanning techniques such as Scan & Paint TPA [16]. The measurement procedure requires measuring under operating conditions and exciting the sound field with a monopole source in two independent stages. Using pressure transducers along with particle velocity sensors (or Microflowns) allows to capture all necessary information for implementing Eq. (6) directly without making any assumption.

2.2 Matrix inversion methods

The large number of multichannel commercial systems based on pressure sensors pushed towards the development of a solution for airborne TPA using only pressure microphones. Matrix inversion was proposed in [17, 18, 19] to indirectly estimate the particle velocity of radiating surfaces or "panels" . The measurement procedure required for estimating pressure contribution of each "panel" is the following:

- A number of "indicator" pressure responses (p_i) are measured close to the radiating surface in operating conditions.

- Near-field transfer functions, between pressures at these indicator positions and volume velocities at the radiating surface, are processed together to calculate the operating volume velocity of the radiating panels.

- The transfer function matrix is measured in a reciprocal way by putting monopole sources at the location of the indicator pressure microphones, and microphones very close to the radiating surface.

Once the data is acquired, the volume velocity of individual panels (particle velocity u_n times panel area A_n) can be calculated such as

$$\begin{bmatrix} u_1 A_1 \\ \cdots \\ u_n A_n \end{bmatrix} = \begin{bmatrix} p_1^{TF}/Q_1 & \cdots & p_1^{TF}/Q_n \\ \vdots & \ddots & \vdots \\ p_i^{TF}/Q_1 & \cdots & p_i^{TF}/Q_n \end{bmatrix}^{-1} \begin{bmatrix} p_1 \\ \cdots \\ p_i \end{bmatrix} \tag{7}$$

The matrix inversion is normally improved by overdetermination, i.e. by taking the number of indicator pressures (p_i) higher than the number of equivalent volume-velocity sources ($u_n A_n$). Next, the expression which is commonly used for relating the pressure at a receiver position with the velocity distribution and transfer functions is

$$p_r = \sum_{n=1}^{N} \frac{p_r^{TF}}{Q_n} u_n A_n \tag{8}$$

where Q_n represents the volume velocity of the monopole source used during the reciprocal transfer path measurements.

Comparing this last expression with the fundamental equation for Airborne TPA given in Eq. (3) one important difference shall be highlighted: one of the terms has been neglected in Eq. (8) . Matrix inversion methods assume that the surface of the structure evaluated can be considered acoustically rigid such that the normal velocity is nearly zero during the transfer path measurements. This argument simplifies the measurement procedure but it is not always suitable. This assumption can only be made for the low frequency region when considering the most common applications of Airborne TPA: car interior noise. In next section this issue is assessed in detail evaluating experimental data.

3 Experimental evaluation

As have been shown, matrix inverse methods and particle velocity-based techniques for Airborne TPA have two different theoretical approaches. First of all, matrix inversion calculates particle velocity indirectly and then disregards one of terms given in the general expression (Eq. (3)). This simplification can be justified assuming that the particle velocity across **S** is very low during the transfer path measurements. This physically implies that all the panels of **S** are acoustically hard. Consequently, Eq. (3) is simplified to obtain an expression (Eq. (8)) that only requires pressure measurements. This assumption disregards several physical effects such as the acoustic absorption of the panels. Typically those effects grow in importance as frequency is increased. The importance of one or other term can be measured by calculating the ratio between both of them across frequency. Figure 4 presents the results found for a car interior test. The given ratio has been calculated using the data of the measurement described in detail in [5]. As can be seen, the basic term used by inverse methods is much higher compared with the neglected term only below 500 Hz. Then, both terms become significant since they carry approximately the same energy. Moreover, it is important to highlight that the neglected term is dominant at some high frequency regions (1 kHz to 1.6 kHz), demonstrating the weakness of the assumption made by inverse methods for high frequency analysis.

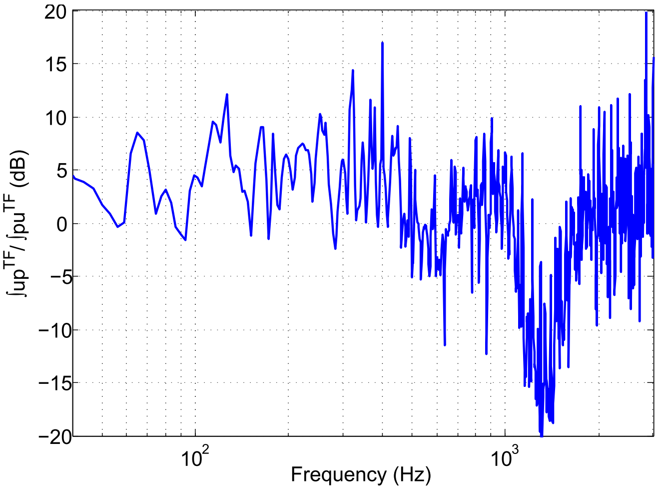

Figure 4: Ratio between the terms involved in Eq. (3)

Furthermore, Figure 5 illustrates how the pressure synthesis changes if the hard-wall assumption is implemented. As could be expected after assessing Figure 4, the pressure reconstruction is fairly good below 1 kHz even disregarding the second term of Eq. (3). However, it is proven to be necessary to take into account the general expression without simplifications in order to achieve a good synthesized pressure for higher frequencies.

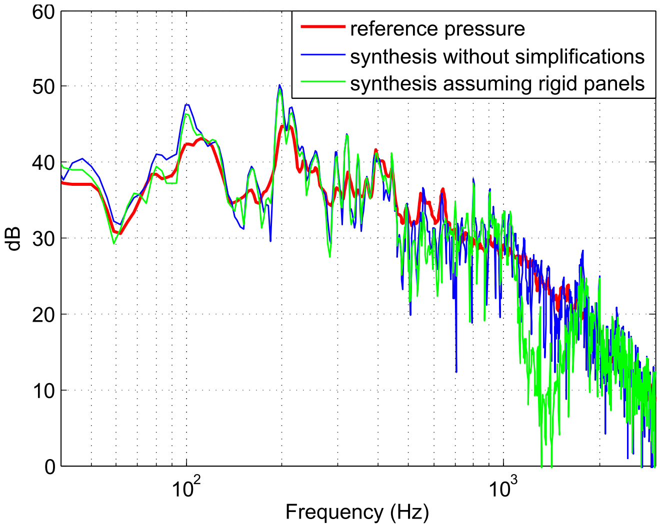

Figure 5: Example of measured reference pressure (red) and synthesized reference pressure assuming rigid panels (green) and without making any assumption (blue) in a car interior.

In addition, matrix inversion methods calculate acoustic particle velocity indirectly, therefore the best achievable estimation would perfectly match the direct measurement results. It implies that any errors derived from the matrix inversion or practical issues would decrease the accuracy of the inverse method.

4 Particle velocity sampling versus inverse methods

Most measurement techniques based on acquiring particle velocity information in the vicinity of a radiating surface suffer mainly from issues related to the discretization of the sound field. The surface vibration distribution is assumed completely characterized by the measurement positions. The presence of leaks or dominant noise sources which are not covered with the measurements would lead to bad spectral estimations since not all the acoustic energy radiated have been considered. Indeed studies have shown [20] that spatial aliasing of the vibration field, due to point sampling, can result in large errors in radiated field estimates. Nonetheless, the use of scanning techniques instead of fixed sensor position allows to increase remarkably the spatial resolution of the method, solving most potential problems of velocity-based methods related with the sampling of the vibrational field.

On the other hand, inverse methods for Airborne TPA strongly depend on the accuracy of the transfer function matrix inversion. The sensitivity to measurement errors can be evaluated by computing the condition number of the matrix. Ill-conditioned matrices (large condition numbers) may result in large error of the predicted particle velocities. This undesired effect can be reduced by using matrix regularization techniques, although they also decrease the accuracy of the inverse, leading to poorer transfer function estimations. Previous studies have shown [21] that it is common to have ill-conditioned matrices in the mid-low frequency range, resulting in large errors in the reference pressure estimation.

Experimental data presented in Section 3 prove the limitations of the inverse approach for estimating the pressure contribution at a reference position. The hard-wall assumption made on the derivation of the inverse method directly biases the results in the high frequency region (see Figure 5). This fact emphasize the lack of robustness of inverse methods for most industrial applications. The required assumption is fairly true at mid-low frequencies but the condition number of the transfer function matrix is usually very high, leading to poor results. In addition, the hard-wall assumption becomes meaningless at higher frequencies, where the transfer function matrix can be inverted accurately.

In summary, particle velocity-based methods, such as the novel reference-related techniques, are more robust and accurate than pressure-based inverse approaches. Velocity-based techniques not only consider a general expression which does not require any assumption of the sound field, but also they do not have errors derived from matrix inversion for indirect velocity estimations.

5 Conclusions

Two widespread methodologies for Airborne Transfer Path Analysis have been derived and evaluated. The theoretical basis of both methods were compared, demonstrating that inverse methods rely on a simplified expression for pressure contribution synthesis. Furthermore, experimental data have illustrated the importance of the assumptions made on the inverse methods, which become dramatically important in high frequencies, where the method does not achieve good results. The use of particle velocity sensors or Microflowns seems to be most suitable solution for direct implementation of Airborne Transfer Path Analysis within a wide frequency range.

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