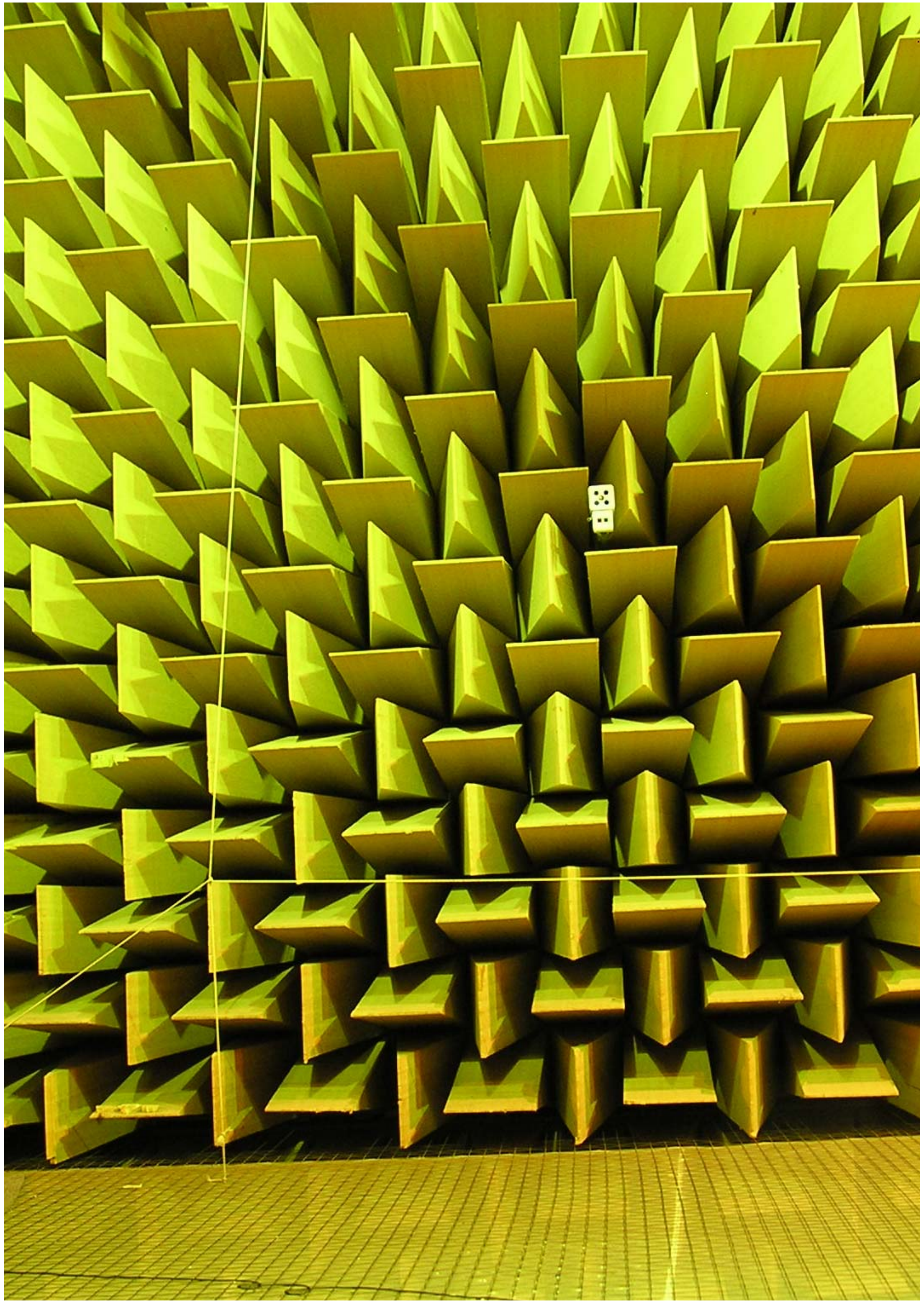

2 Sound & Vibration

Fig. 2.1 (previous page): The sound absorbing wall of the large anechoic room of the Danish Technical University. The 'floor' is a metal net with below similar absorbers as the wall.

2.1 Introduction

The acoustic technical world is divided in two disciplines: sound and vibrations. To put is simple: "sound" is what one can hear and "vibrations" is what can feel if a structure vibrates. Vibrating objects usually produce sound so the two disciplines have definitely some overlap. However the measurement tools and theory behind the subjects is very different and this is probably why the acoustic world is divided in two disciplines.

This chapter starts with theory on sound. Three matters are discussed. What is sound; how can one detect sound and what acoustic measurements can be performed? Definitions of acoustic quantities are given. Relations are shown between sound pressure and particle velocity. Operation principles of sound pressure, pressure-gradient and particle velocity microphones are presented. Measurements that require both sound pressure as particle velocity like sound intensity, sound energy and acoustic impedance measurements complete this part.

After this the subject 'vibrations' is considered. Vibrations are movements in solids and these are usually measured with other type of sensors than the sensors used for measuring sound. The two most common ones are accelerometers and lasers. Accelerometers are sensors that must be attached to the vibrating object and are sensitive for the acceleration of the object rather than the movement. A laser is a non-contact method to measure the movement of the object.

2.2 What is sound

Very simply, sound can be observed as a propagating vibration in any substance. This substance can be air, water, wood, or any other material: in fact only in vacuum sound can not propagate. Here, sound is observed where it is perceived most: in the air.

Sound is produced by a source, for example a firecracker, it travels through the air with a certain velocity (which is about 340m/s or 1225km/h) and can be reflected perhaps a few times by a wall or the ground. An ear or microphone detects it. After perceiving the sound of the firecracker one can distinguish its origin. Besides that, it can be said to be a loud bang and perhaps to be an annoying experience.

This annoying experience, or the quantification "loud", is a subjective interpretation of sound. If large amounts of people are annoyed by a certain distress factor, regulations will follow. In this case sound measurements to quantify the amount of noise should be performed. Questions like "was it really a loud bang or was it somebody who didn't like firecrackers?" should be answered in a proper way because of the subjectivity of perceiving sound.

The human ear is very sensitive for small pressure variations in the air. Not all frequencies are in a similar way similar. The human ear is most sensitive for pressure variations of about 3kHz (corresponding to 3000 variations per second). For frequencies of 30Hz the ear is about 200 times less sensitive. The static air pressure is about hundred thousand Pascal (Pa), a pressure variation of one Pascal is perceived as very loud at 1kHz, but as a soft tone at 20Hz, see also Fig. 2.22.

Sound can be understood as variations of the static air pressure *and* as a velocity of air particles: particle velocity. The following may explain this.

Pressure is defined as force per unit area. This force is caused by an amount of air particles at a certain position. So variations in the (sound) pressure is influenced by variations in the number of air particles. If the number of particles in a certain volume changes during time, there must be a particle motion towards that volume and backwards. The velocity of the movement of these air particles is called particle velocity. Therefore sound consists always of two parts: the compression and decompression of air, sound pressure, and the movement of the air: the particle velocity.

For the complete quantification of a sound field, both sound pressure as particle velocity should be measured.

In an electrical analogy, sound pressure corresponds to an electric voltage and particle velocity to electric current. The amount of power is quantified in Watts. Multiplication of voltage and current results in electrical power and analogously, sound pressure times particle velocity is associated with sound power in the acoustical domain.

In general the particle velocity is associated with the cause of an acoustic event and the sound pressure is associated with the result. If for example a loudspeaker is generating a sound wave in a room. Then the particle velocity measured close to the loudspeaker quantifies the level of vibration (the cause of the sound wave). The sound pressure measure in the room quantifies the amount of sound that is heard. If the walls of the room are hard reflecting, the measured sound pressure will be higher than if the walls of the room were fully sound absorbing. The particle velocity measured close to the loudspeaker is nit affected by the acoustic properties of the walls.

Sound pressure, affected by the number of air particles at a certain position, does not have a direction; sound pressure is a scalar quantity. Contrary, the velocity of air particles does have a certain direction: particle velocity is a vector quantity.

To find out the direction of propagation of a sound wave, sound pressure and particle velocity must be measured simultaneously at one location. The product of both provides information about the direction of the sound wave.

The human ear detects sound pressure only. So how is it possible that we can find out where sound is coming from? This has to do with the shape of the ear and with the fact that we have two ears. Actually this is an example of the most elementary array technology. A human head can be seen as two pressure microphones (P_{left}&P_{right}) separated 20cm by a solid (the head), see Fig. 2.2 Here the sound source localisation will be explained very briefly, in [14] theory is presented in more detail.

With the use of both ears one can detect the sound level (difference) and one can detect the time difference between perceiving sound in one ear and the other. If it is assumed that the sound source is located left forward (Fig. 2.2A) and the frequency is not too high, the sound must travel longer to the right ear than to the left ($L_{left}<L_{right}$). Since low frequency sound waves have very long wavelengths, the head is considered a small obstacle. The sound wave tends to bend around small obstacles and therefore the intensity

difference between the two ears is little. By the use of this time delay (or more specific, the phase shift) the source is localised.

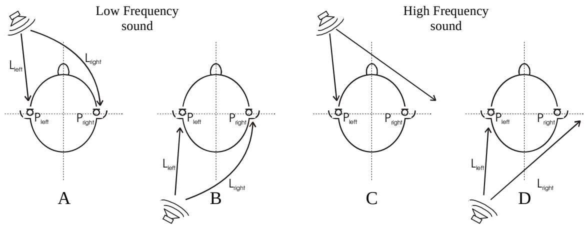

Fig. 2.2: Schematic head perceiving sound and sound waves, represented very basically.

Very simply explained, if the sound source is located left backward (Fig. 2.2B) one detects the same phase shift: the shape of the ears makes the discrimination possible between forward and backward. (Head movements do have also something to do with the source localisation).

Since for higher frequencies the wavelength becomes shorter than the separation of both ears and the phase shift method becomes invalid. Because the head now is larger than the wavelength it becomes an obstacle for sound waves. In situation Fig. 2.2C the left ear will perceive more sound than the right ear: the sound source location is localised by sound level difference. Again, if the sound source is located left backward (Fig. 2.2D) one detects the level difference. The shape of the ears is used again to discriminate between forward and backward.

As stated, the ear can detect sound pressure only and it is impossible to perceive particle velocity. How about acoustic sensors? Well, there are three types of sensors. First, sound pressure sensors, like the ear. Second, sound pressure gradient microphones; this type of acoustic sensor is sensitive for the pressure difference over a distance (like the human head for low frequencies: two pressure sensors separated by a certain distance). Third, there is only one type of particle velocity sensitive microphone: the Microflown.

Sound pressure microphones detect pressure variations by deflection of a very thin membrane due to the sound pressure difference inside and outside the sensor. It functions similar to the ear. A pressure gradient microphone (also called a velocity microphone) measures the difference in the sound pressure at two spaced places. It has, in a limited bandwidth, a close resemblance to the particle velocity.

To get a better understanding of the relation between the sound pressure, sound pressure gradient and the particle velocity, the wave equation will be examined.

2.3 An introduction to acoustics

In this paragraph concepts like sound pressure, particle velocity and their relationship are described. For simple cases like plane waves, solving the one-dimensional wave equation yields the relation between the sound pressure and particle velocity. If the situation becomes more complex, like for instance sound fields near a sound source, the wave equation must be solved three-dimensionally. Only the one-dimensional wave equation is solved here.

Once the sound pressure and the particle velocity are known, the product and the ratio of both can be calculated. The product (associated with the sound intensity) provides information on the amount of acoustic noise and the ratio (the specific acoustic impedance) provides information on the nature of the sound field and can for instance be used to determine the reflection coefficient of acoustic absorbent materials.

The sound intensity measurement, which is associated with the product of sound pressure and particle velocity, is a main application of the Microflown. Nowadays this measurement is performed with a probe that consists of two closely spaced pressure microphones, a p-p probe. The sound pressure is determined with the sum of the two microphones; the particle velocity is calculated from their differential signal. The one-dimensional wave equation should be understood to comprehend this.

The one-dimensional wave equation

As sound waves travel through matter, each molecule hits another and returns to its original position. The result is that regions of the medium become alternately more dense, being condensations, and less dense, called rarefactions.

Now, observe a small volume ($S \cdot dx$) of air, see Fig. 2.3. Inside the volume there is a certain number of air particles, which can be associated with the air density. At place x particles enter the volume with a certain velocity $u(x,t)$ and particles leave the volume (at place $x+dx$) with a velocity $u(x+dx,t)$: the sound wave is travelling in the x-direction only.

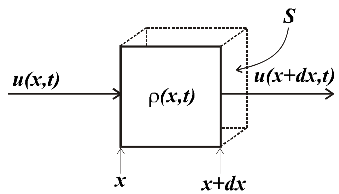

Fig. 2.3: A volume of air through which an acoustic disturbance passes.

The amount of mass that enters the volume equals: $S{\cdot}\rho(x,t)\ u(x,t)$. The amount that leaves the volume: $S{\cdot}\rho\ (x+dx,t)\ u(x+dx,t)$. Using ρ as the air density. The net rate of mass inflow therefore equals:

$$S\rho(x,t)u(x,t) - S\rho(x+dx,t)u(x+dx,t) = -S\frac{\partial}{\partial x}\big(\rho(x,t)u(x,t)\big)dx \tag{2.1}$$

The principle of mass conservation requires that this net inflow must be balanced by an increase in mass of the element:

$$-S\frac{\partial}{\partial x}\rho(x,t)u(x,t)dx = \frac{d}{dt}S\cdot dx\cdot\rho(x,t) \tag{2.2}$$

The density inside the volume can be described with $\rho(x,t)=\rho_0+\Delta\rho(x,t)$, a constant density ρ_0 plus a time varying part $\Delta\rho(x,t)$. The product of the density and the particle velocity equals: $\rho(x,t){\cdot}u(x,t)=u(x,t)\rho_0+\Delta\rho(x,t)\ u(x,t)$. If the density variation can be considered very small compared to the static density, the latter equation can be simplified into: $\rho(x,t){\cdot}u(x,t)\approx u(x,t)\rho_0$. The simplified version of Eq. (2.2) is known as the linearised equation of mass conservation and is only valid if the variation in density is small compared to the density itself:

$$\boxed{\frac{\partial}{\partial t}\Delta\rho(x,t) + \rho_0\frac{\partial}{\partial x}u(x,t) = 0} \tag{2.3}$$

The linearised equation of mass conservation can be used to calculate the sound pressure component of a sound wave if the particle velocity is measured at two closely spaced places. This will be clear when Eq. (2.3) is integrated with respect to time:

$$\Delta\rho(x,t) = -\rho_0\int\frac{\partial}{\partial x}u(x,t)dt \approx -\rho_0\int\frac{u(x+\Delta x,t)-u(x,t)}{\Delta x}dt \tag{2.4}$$

For “normal” sound levels (less than 135dB) it is assumed that the increase in pressure and increase in density are linearly related by $p(t,x)=c^2\ \Delta\rho(t,x)$. And thus:

$$p(x,t) = -\frac{\rho_0 c^2}{\Delta x}\int u(x+\Delta x,t)-u(x,t)dt \tag{2.5}$$

With the use of two closely spaced (spacing Δx) particle velocity probes (the separation distance must be much smaller than the wavelength) the (one-dimensional) sound pressure can be determined. (In fact 6 Microflowns are needed to determine the sound pressure in a three dimensional sound field).

The same small volume ($S{\cdot}dx$) of air is observed again, see Fig. 2.3. Now, for this volume the Newton’s second law, force equals mass times acceleration, (**F**=m**a**) is solved.

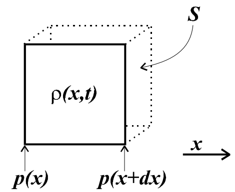

Fig. 2.4: A volume of air through which an acoustic disturbance passes.

For the acceleration (the time derivative of the velocity) of the particles in the volume element $S{\cdot}dx$ can be written:

$$\frac{d}{dt}u(x,t) = \frac{\partial u(x,t)}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial}{\partial t}u(x,t) = \frac{\partial u(x,t)}{\partial x}u(x,t) + \frac{\partial}{\partial t}u(x,t) \tag{2.6}$$

For a harmonic particle velocity $u(x,t){\cdot}\partial u(x,t)/\partial x$ is much (u/c) smaller then $\partial u(x,t)/\partial t$ and therefore Eq. (2.6) will simplify in to:

$$\frac{du(x,t)}{dt} \approx \frac{\partial u(x,t)}{\partial t} \tag{2.7}$$

The mass of the volume is given by: $S{\cdot}dx{\cdot}\rho_0$. The net force in the x-direction equals: $S{\cdot}p(x,t)-S{\cdot}p(x+dx,t)$. This net force equals the product of mass times and acceleration of the volume:

$$S\cdot p(x,t) - S\cdot p(x+dx,t) = \underbrace{-\frac{\partial}{\partial x}p(x,t)\cdot S\cdot dx}_{F} = \underbrace{S\cdot dx\cdot\rho_0}_{m}\underbrace{\frac{\partial}{\partial t}u(x,t)}_{a} \tag{2.8}$$

The latter can be simplified into:

$$\boxed{\rho_0\frac{\partial}{\partial t}u(x,t) + \frac{\partial}{\partial x}p(x,t) = 0} \tag{2.9}$$

This relationship is known as the linearised equation of momentum conservation and is only valid if the particle velocity u is small compared to the velocity of sound *c*.

Analogous to the calculation of sound pressure using the *u-u* method, the linearised equation of momentum conservation can be applied to deduce an expression for the particle velocity (by use of two closely spaced pressure microphones, called the *p-p* method):

$$u(x,t) = -\frac{1}{\rho_0}\int\frac{d}{dx}p(x,t)dt \approx -\frac{1}{\Delta x\cdot\rho_0}\int p(x+\Delta x,t)-p(x,t)dt \tag{2.10}$$

If the separation distance of the microphones is much smaller than the wavelength, the particle velocity can be determined in one direction. This is called the p-p method and will be dealt with in more detail at the end of this chapter (section "Measuring sound intensity") and in chapter 5: 'sound intensity'.

Differentiating Eq. (2.3) with respect to time and Eq. (2.9) with respect to x yields:

$$\frac{\partial^2 p(x,t)}{\partial x^2} - \frac{\partial^2 \Delta\rho(x,t)}{\partial t^2} = 0 \tag{2.11}$$

The sound pressure and density variations are linearly related: ($p(x,t)=c^2 \Delta\rho(x,t)$). By substituting this in Eq. (2.11), the one-dimensional wave equation is achieved:

$$\frac{\partial^2 p(t,x)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p(t,x)}{\partial t^2} = 0 \tag{2.12}$$

This wave equation must be satisfied by acoustic waves propagating in the medium. However, in order to find solutions to this equation, the form that the sound field must take must be specified in more detail. Boundary conditions must be specified in order to find the form of the sound field.

2.4 Definitions

In acoustics a particle is defined as a volume of air that is small compared to the dimensions of our measuring device or the wave length of the acoustic wave and large compared to the molecular mean free path ($5\cdot10^{-8}$m) [13]. The minimal wavelength in air being about 17 millimetres, the measuring device can be for instance an eardrum (5 millimetre) or a Microflown (100 micrometer). A particle may be interpreted as a cube of air with dimensions of $1\mu m \times 1\mu m \times 1\mu m$.

The sound pressure, *p(t)*, is the incremental change from the static pressure at a given instant caused by the presence of the sound wave. Pressure can be associated with the number of air particles. The particle velocity ***u****(t)*, is the velocity of a given infinitesimal part of the medium (the above mentioned particle) at a given instant due to a sound wave only. Note that sound pressure is a scalar and particle velocity is a vector quantity.

The human ear is sensitive for sound pressure in the frequency, *f*, range from 20Hz up to 20kHz. Sound waves with frequencies higher than 20kHz are called ultra sound and sound waves with frequencies lower than 20Hz are called infra sound. Low frequency sound waves have frequencies lower than 200Hz.

The wavelength λ is given by the speed of propagation of the wave, *c,* divided by the frequency of vibration:

$$\lambda = \frac{c}{f}[m] \tag{2.13}$$

In air acoustic wavelengths extend over a range from 17 millimetres (f=20kHz) to 17 meters (f=20Hz).

The specific acoustic impedance at a point in a sound field is defined as the ratio of the complex amplitude of an individual frequency component of sound pressure at that point, to the complex amplitude of the associated component of particle velocity.
It is a complex number (not a vector), giving the magnitude and phase of the ratio of the individual frequency components. It can be compared to the electrical resistance in the electric domain.
The acoustic impedance can be e.g. be used to determine the reflection coefficient of acoustic absorbing materials or to find out if a source is an effective radiator.

The specific acoustic impedance is given by:

$$Z_s = \frac{p}{u}[Ns/m^3] \tag{2.14}$$

Note that the specific acoustic impedance is a complex figure and not a vector quantity.

The measurement of the specific acoustic impedance is used for example to determine the sound absorbing characteristics of materials like glass wool or foams. The reflection coefficient can be calculated from the impedance, see further chapter 6: 'acoustic impedance'.

The characteristic impedance is the ratio of the effective sound pressure at a given point to the effective particle velocity at that point in a free, plane, progressive sound wave. It is equal to the product of the density ρ, of the medium (air) times the speed of sound *c* in the medium ($Z=\rho c\approx 435 Nsm^{-3}$). A plane wave is an acoustic disturbance in which the pressure is uniform in the direction normal to the direction of propagation. When measured far from the acoustic source in the free field one obtains a plane wave. In anechoic rooms free field conditions can be found.

The sound intensity in a specified direction at a point is the average rate at which sound energy is transmitted through a unit area perpendicular to the specified direction at the point considered. It is thus a vector quantity; defined as the time averaged product of the sound pressure (a scalar quantity, *p*) and the corresponding particle velocity (a vector quantity, **u**) at the same position.

$$\boldsymbol{I}[W/m^2] = \lim_{T\to\infty}\frac{1}{T}\int_0^T \boldsymbol{u}(t)\cdot p(t)dt \tag{2.15}$$

Note: the sound intensity is not depending on time.

Sound energy density *E* is defined as the sum of potential energy *U* and the kinetic energy K, both per unit volume. The potential energy U is associated with the sound pressure and is given by:

$$U = \frac{|p|^2}{2\rho_0 c^2} \tag{2.16}$$

The kinetic energy density *K* is associated with the particle velocity and is given by:

$$K = \tfrac{1}{2}\rho_0 |u|^2 \tag{2.17}$$

Since nowadays apart from the sound pressure, particle velocity can be determined instantly, acoustic quantities as sound intensity, acoustic impedance and sound energy density can be measured three dimensional, instantaneous and on one position in space.

Sound intensity is associated with the product of sound pressure and particle velocity and quantifies the amount of sound energy that propagates. The specific acoustic impedance is related with the ratio of both, and is a useful quantity to determine for example the reflection or absorption of matter. Sound energy is associated with the sum of sound pressure and particle velocity.
Sound energy density quantifies how much energy is “stored” in an acoustic wave, sound intensity quantifies how much sound energy is transported and specific acoustic impedance quantifies the possibility for sound energy to be transported.

Sound intensity is the rate of which sound energy is transported. For a one dimensional case the following expression is valid:

$$\frac{\partial}{\partial t}E + \frac{\partial}{\partial x}I = 0 \tag{2.18}$$

For most purposes, the absolute numbers obtained from measurements have little significance themselves; they are almost always compared to a reference, and are usually quoted as: levels "re. …" that reference. As in many physical sciences, the units and reference levels are derived from common observed phenomena. The zero-level of sound pressure, for example, is not a physical zero (that is the absence of sound pressure); rather, it is something of an average "threshold of hearing" at about 1kHz for humans. The physical pressure associated with this level is very small (20 micro Pascal). About one million times greater sound pressure tends to cause considerable discomfort. Since the sound intensity generally varies with the square of the sound pressure, this (audio) range represents a ratio of one trillion (10^{12}) to one in energy!

The most known property is the sound pressure level (SPL) which is given in decibels. It is defined as twenty times the logarithm to the base ten of the ratio of the measured sound pressure of this sound to a reference effective sound pressure of 20 micro Pascal.

$$SPL = 20Log_{10}\frac{p}{p_{ref.}}(re. = 20\mu Pa)[dB] \tag{2.19}$$

The reference 20 micro Pascal is chosen because a human can just hear this amount of sound at one kilohertz. In underwater acoustics a sound pressure reference of 1μPa is used.

The particle velocity level, also given in decibels, has a reference effective particle velocity of 50 nano-meter per second. The origin of this reference is the reference pressure level divided by the characteristic impedance (20μPa/ρc≈46nm/s). So it is the particle velocity corresponding to the reference sound pressure in a free, plane, progressive sound wave.

It can be argued that the reference should be 20μPa/ρc≈46nm/s in stead of 50nm/s. However the characteristic impedance (ρc) is (amongst others) temperature dependent so if the reference is chosen 20μPa/ρc, the reference level would also be. This is rather inconvenient but once the reference is fixed on 50nm/s, sound pressure levels and particle velocity levels may not be the same in a free progressive wave.

So by definition the particle velocity level (PVL) equals:

$$PVL = 20Log_{10}\frac{u}{u_{ref}}(re. = 50nm \cdot s^{-1})[dB] \tag{2.20}$$

The sound intensity level has a reference intensity of one pico-Watt.

$$\boldsymbol{S}IL = 10Log_{10}\frac{I}{I_{ref}}(re. = 1pW)[dB] \tag{2.21}$$

The reference of 1pW is because 20μPa×50nm/s=1pW.

Various values of the magnitudes mentioned quantities are given in Table 2.1. In this case it is assumed that the specific acoustic impedance equals the characteristic impedance so that the values of PVL and SIL are equal to the SPL.

Table 2.1: Some acoustic levels

Description	level [dB]	p [Pa]	u [m/s]	I [W/m2]
Artillery fire, airplane	140	200	0.45	100
Rock concert	120	20	0.045	1
Reference level	94	1	2.5×10^{-3}	2.5×10^{-3}
Vacuum cleaner	80	0.2	4.5×10^{-4}	1×10^{-4}
Conversation, face-to-face	60	0.02	4.5×10^{-5}	1×10^{-6}
Whispering	40	2×10^{-3}	4.5×10^{-6}	1×10^{-8}
Inside empty theatre	20	2×10^{-4}	4.5×10^{-7}	1×10^{-10}
Threshold of hearing	0	2×10^{-5}	4.5×10^{-8}	1×10^{-12}

2.5 Sound fields

There are some specific sound fields that one should know to make acoustic problems easier to understand. Practical sound fields can be considered as a combination of these fields.

As long as the acoustic phenomena are linear (i.e. the sound levels are below 135dB) separate sound fields do not interact.

The far field (plane waves)

The far field is a sound field far from a sound source in an environment where no reflections exist. The sound pressure and particle velocity are in phase or out of phase and their ratio is given by the characteristic impedance $Z=p/u=\rho c$.

The amplitude of the sound pressure and particle velocity is depending inversely on the distance to the source. If the distance to the source doubles, the amplitude of sound pressure and particle velocity will be half.

Only under free field conditions (far from the source where the sound waves are plain and no reflections are present), both sound pressure and particle velocity are completely in (A) or out phase (B) and the measured sound levels will be of the same magnitude, see Fig. 2.5.

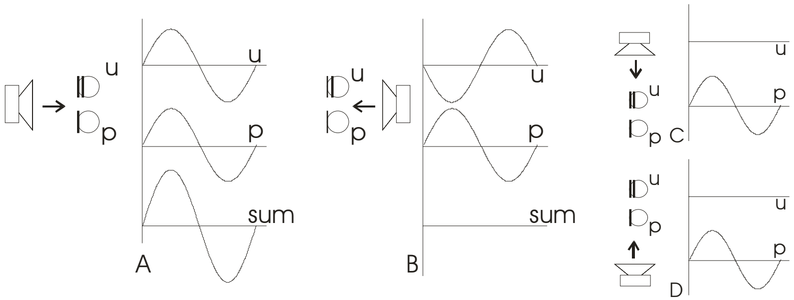

Fig. 2.5: output of a microphone and Microflown for plane sound waves.

Reflections

Reflections occur if in the path of the sound wave the acoustic impedance alters. Normally reflections are caused due to the presence of a material but also temperature variation in the air temperature can cause a reflection.

As an example an acoustic burst is generated in a 40 meters long tube with a loudspeaker at the left side and the sound is reflected at the right side. As can be seen, the sound pressure and particle velocity are in phase in the forward travelling wave. The reflected sound wave is out of phase,

the pressure signal has the same phase as the forward travelling wave and the reflected sound wave is reduced in amplitude due to damping.

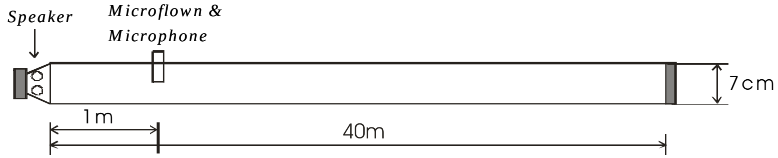

Fig. 2.6: The 40m long tube measurement set up is used to demonstrate the effect of reflections.

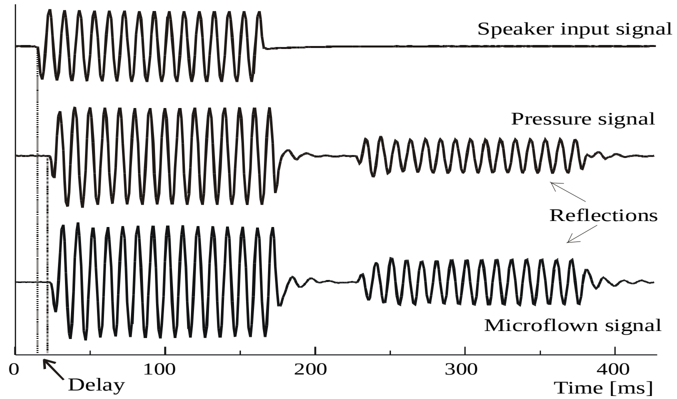

Fig. 2.7: A burst measurement in a 40 meters long tube.

A short sine wave burst is not the signal that is most common. Normally signals are generated continually and the sound field will be a summation of the original signal with the reflections.

Compared to the original signal in general a reflection has: 1) an opposite phase between the sound pressure and particle velocity; 2) the sound pressure has the same phase; 3) the amplitudes of sound pressure and particle velocity are reduced.

As an example of the difference between sound pressure and particle velocity, a burst of two sound waves are generated in a tube in opposite directions, see Fig. 2.8.

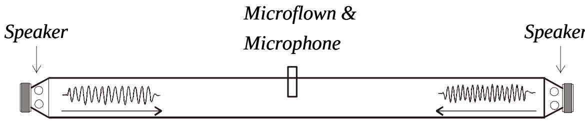

Fig. 2.8: Measurement set-up to demonstrate the difference between sound pressure and particle velocity.

One speaker generates a 150Hz burst and the other a 200Hz burst. The Microflown and microphone are situated in the middle measuring both sound waves travelling in opposite directions (upper two curves in Fig. 2.5).

As can be seen, the output of both microphones is different. If the two signals are subtracted the result will be the leftwards-travelling wave and the sum will result in the rightwards-travelling wave. This is because the particle velocity has a phase shift compared to the sound pressure if the direction is altered from forward to backward, see Fig. 2.6. If both Microflown as microphone have the same sensitivity, the sum of both sensors will result in a uni-directional sensor assembly.

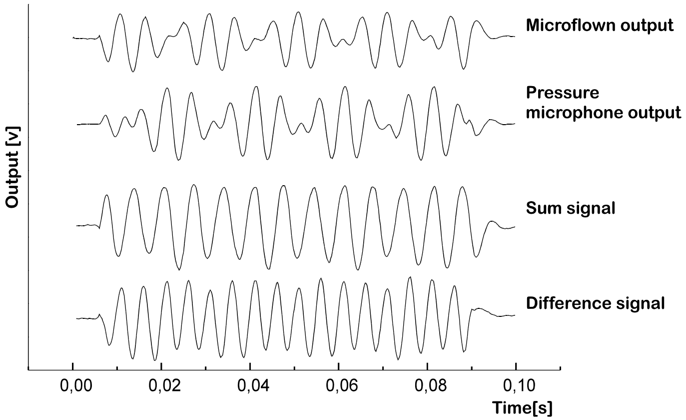

Fig. 2.9: Measured signals of a Microflown and a microphone; their sum and difference.

The near field

As an example the wave equation will be solved for a spherical sound field radiated from a point source. One can observe the sound intensity to find a suitable function of the sound field. Since the surface of the sphere around the point source increases with the square of the radial distance the sound intensity, the sound power per unit area, decreases with $1/r^2$. The sound power is a product of the sound pressure and the particle velocity. This product is a function of $1/r^2$ and the sound pressure and the particle

velocity are linear related, therefore they will both be a function of $1/r$. The radiation from the point source is spherically symmetric and the wave equation for spherically symmetric wave propagation is given by (see appendix 3):

$$\frac{\partial^2(r\cdot p)}{\partial r^2}-\frac{1}{c^2}\frac{\partial^2(r\cdot p)}{\partial t^2}=0 \tag{2.22}$$

Since the function $r{\cdot}p$ should obey the wave equation Eq. (2.22), of which the solution is of the form $Ae^{i(kx\pm\omega t)}$, ($k=\omega/c$ known as the wave number) the pressure p should be a harmonic outwards-travelling wave proportional to $1/r$:

$$p(r,t)=\mathrm{Re}\left\{Ae^{i(wt-kr)}\cdot r^{-1}\right\} \tag{2.23}$$

Substitution of Eq. (2.23) in Eq. (2.22) shows that this function of the sound pressure obeys the wave equation. The linearised equation of momentum conservation can be used to find the particle velocity that is associated with the sound pressure:

$$u(r,t)=\frac{-1}{\rho_0}\int\frac{d}{dr}\,p(r,t)dt=\frac{A}{i\omega\rho_0}\left(\frac{1}{r}+ik\right)\frac{e^{i(\omega t-kr)}}{r} \tag{2.24}$$

The specific acoustic impedance as a function of the radial distance:

$$z(r)=\rho_0 c\ \left(\frac{ikr}{ikr+1}\right) \tag{2.25}$$

Eq. (2.25) shows that, if the distance to the sound source is small, for spherical waves the specific acoustic impedance is proportional to k, and thus for the frequency. In practice this means that relative close to the source the acoustic impedance is lower than ρc and frequency dependent. At lower frequencies the particle velocity level will be larger than the sound pressure level. This is sometimes called the nearfield effect.

Very near field

In this paragraph theory is presented that predicts the region of the very near field and how sound pressure and particle velocity behave in this region [6], [7]. A sound wave of frequency f can be described by the acoustic potential $\varphi(\mathbf{r})$ obeying the Helmholtz equation:

$$\Delta\varphi+k^2\varphi=0 \tag{2.26}$$

Here Δ is the Laplace operator, $k=2\pi/\lambda=2\pi f/c$ is the wave number, λ is the wavelength. To describe the sound field from some source, which is a vibrating surface, this equation should be solved with the boundary conditions:

$$\frac{\partial \varphi}{\partial n} = u_n \text{ on the surface,}$$
$$\varphi \propto \frac{\exp(ik_0 r)}{r} \text{ at infinity } r \to \infty, \quad (2.27)$$

where ∂/∂n is the derivative normal to the surface, u_n is the normal component of the velocity, in general, a function of the point on the vibrating surface. The observable acoustic values, sound pressure p and particle velocity $\boldsymbol{u}$, are connected with the potential in the following way:

$$u = grad\ \ \varphi\ \ ,\quad p = -i\omega\rho\varphi \quad (2.28)$$

where ρ is the density of the background medium (air).

Suppose that the vibrating surface can be considered as flat on some lateral length scale L. One can choose the x-y plane on the surface then the z direction will coincide with the normal to the surface. In the direct vicinity of the surface the potential can be expanded in the power series in z:

$$\varphi(x,y,z) \approx \varphi_0(x,y) + \varphi_1(x,y)z + \frac{1}{2}\varphi_2(x,y)z^2 + O(z^3) \quad (2.29)$$

The functions $\varphi_{0,1,2}$ are connected with the velocities on the surface. From first the boundary condition (2.27) it follows that:

$$\varphi_1(x,y) = u_z(x,y,0) \equiv u_z \quad (2.30)$$

It means that measured particle velocity very close the surface coincides with the surface velocity (this is verified by measurements, see below). Taking the second derivative with z from Eq. (2.29), one finds:

$$\varphi_2(x,y) = \left(\frac{\partial u_z}{\partial z}\right)^S \quad (2.31)$$

Therefore, the expansion Eq. (2.29) can be written as:

$$\varphi(x,y,z) \approx \varphi_0 + u_z^S z + \frac{1}{2}\left(\frac{\partial u_z}{\partial z}\right)^S z^2 \quad (2.32)$$

Let us insert now the potential (2.32) in the Helmholtz equation (2.26) and take the limit $z \to 0$. We will find:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\varphi_0 + k^2\varphi_0 + \left(\frac{\partial u_z}{\partial z}\right)^S = 0 \quad (2.33)$$

Suppose that the surface vibration can be described by some spatial wavelength L. It means that the surface vibrations can be represented by a

harmonic in space function like $\sin(2\pi x/L)$ and similar for y-direction. The first two terms in Eq. (2.33) then can be estimated as:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\varphi_0 \sim \left(\frac{2\pi}{L}\right)^2 \varphi_0, \quad k^2\varphi_0 = \left(\frac{2\pi}{\lambda}\right)^2 \varphi_0 \quad (2.34)$$

Typical situation in acoustic is that the vibrating body radiates the sound waves with the wavelength:

$$\lambda >> L \quad (2.35)$$

Then, according to (2.34), the second term in Eq. (2.26) is small in comparison with the first one and in the vicinity of the body the Helmholtz equation (2.26) is reduced to the Laplace equation:

$$\Delta\varphi = 0 \quad (2.36)$$

Equation (2.36) is the equation of uncompressible fluid that means that in this approximation the pressure level (dB) is negligible compared to the velocity level. The normal velocity in this range coincides with velocity of the vibrating surface.

Physically it means that nearby the surface the fluid can be considered as uncompressible and for this reason the sound pressure level (dB) is small compared to the particle velocity level.

The latter has to be clearly explained. The sound waves are the compression-decompression waves and if Eq. (2.36) would be exact there would be no sound pressure at all. In reality this equation is approximate and the pressure still finite but it is suppressed.

It shows that the sound pressure level is suppressed in comparison with the particle velocity level by the factor L/λ (the ratio of the spatial wavelength L of the source and the wavelength) and the phase between the particle velocity and sound pressure is shifted to 90 degrees.

Eq. (2.36) is true if the normal distance $r_n = z$ to the vibrating surface is small in comparison with the size L and that the wavelength λ is larger than the vibrating surface L.

$$r_n << \frac{L}{2\pi} << \frac{\lambda}{2\pi} = \frac{c}{2\pi f} \quad (2.37)$$

The condition (2.37) can be named by the condition of the ***very near field***. In this range the normal component of the velocity coincides with that for the vibrating surface but there are no restrictions on the lateral components of the velocity, which can be distributed in some way along the surface. In the table below the properties summarized.

Region	Condition	u [m/s]	p [Pa]	Phase [deg]
Very near field	$r_n << \frac{L}{2\pi} << \frac{\lambda}{2\pi}$	u(r_n) ≈ constant U(f) = constant	p(r_n) ≈ constant p(f) ~ f	80-90
Near field	$\frac{L}{2\pi} << r_n << \frac{\lambda}{2\pi}$	$u(r_n) \sim r^{-2}$	$P(r_n) \sim r^{-1}$	80-10
far field	$r_n >> \frac{\lambda}{2\pi}$	$u(r_n) \sim r^{-1}$	$p(r_n) \sim r^{-1}$	0-10

Where r_n is the normal distance to the source, L is the typical size of the source and λ is the wavelength of the sound wave.

See further chapter 7: 'Vibration measurements'.

Evanescent sound waves

If the wavelength of the bending waves in the plate is shorter than the acoustic wavelength, then the acoustic waves do not propagate into the fluid. In such cases 'evanescent' waves occur where the pressure amplitude and the normal particle velocity decreases exponentially with distance from the plate. These sound waves carry no energy.

The particle velocity induced by a vibrating panel when the travelling wave speed in the panel is less than the wave speed in the fluid. The amplitude of the particle motion decreases exponentially with increasing distance from the boundary. The particles move in an ellipse.

This effect can be understood in the following way. If the wavelength in air is longer than the bending waves in the plate, the waves in the plate create an infinite number of acoustic shortcuts where the air particles just are 'pumped around'. If the wavelength in air is shorter than the bending waves in the plate the air particles that are 'pushed forward' can not reach the part of the plate where the particles are 'sucked back'. In this case an acoustic wave can be created that propagates.

Coincidence frequency

The speed of sound in air is almost constant but the speed of sound in (sheet) materials varies with frequency. Because of this, there is always a frequency where the wavelength in the air is the same as the wavelength in the material. This is called the coincidence frequency.

If sound waves travel over a plate at the coincidence frequency it is very easy for the plate to start vibrating in at the same frequency. If this happened, the sound wave is 'coupled in', i.e. the sound energy of the sound wave in the air is transferred into the plate.

Once the sound wave is transferred into the plate and the plate is now vibrating at the coincidence frequency, at the other side of the plate the air will vibrate with the same wavelength and the sound wave is 'coupled out',

i.e. the sound energy of the sound wave is transferred into the air behind the plate.

As can be seen a plate that is normally reflecting the sound waves will be 'acoustic transparent' at the coincidence frequency.

This effect is noticed best for lateral sound waves (so sound waves not normal to the plate).

Standing waves

A pure standing wave can be generated in for instance a standing wave tube. In such tube, a rigidly terminated tube with rigged side walls, the sound wave can only travel in one dimension and all the sound is reflected at the end of the tube.

If a p-**u** probe is positioned in the tube at a distance *(l-x)* to the end of the rigid end the sound pressure and the particle velocity can be calculated, see Fig. 2.10.

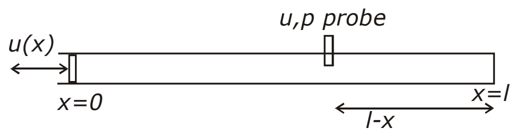

Fig. 2.10: A standing wave tube that is rigidly terminated at x=l and in which the fluid is driven by a loudspeaker at x=0.

For harmonic excitation, one solution that will satisfy the wave equation is given by the superposition of the complex pressures associated with a positive travelling and a negative travelling plane wave. In general the pressure is given by:

$$p(x) = Ae^{-ikx} + Be^{ikx} \tag{2.38}$$

Where *A* and *B* are arbitrary complex numbers which represent the amplitude and relative phases of waves travelling in the positive and negative *x* direction respectively, *k* is known as the wave number and equals $k=\omega/c$ ($\omega=2\pi f$ using *f* as the frequency).

The particle velocity associated with this pressure wave can be found using the linearised momentum equation ($\partial p(x)/\partial x=-i\omega\rho u(x)$).

$$u(x) = \frac{Ae^{-ikx}}{\rho c} - \frac{Be^{ikx}}{\rho c} \tag{2.39}$$

The values of the complex constants *A* and *B* are solved knowing that the particle velocity is zero at *x=l*: $Ae^{-ikl} = Be^{ikl}$ and at the left hand side of the tube the particle velocity is given by the movement of the piston *u(0)=U*: $\rho cU = A - B$. The sound pressure and particle velocity at any place in the tube are given by:

$$p(x) = -i\rho cU \frac{\cos(k(l-x))}{\sin(kl)} \qquad u(x) = U\frac{\sin(k(l-x))}{\sin(kl)} \tag{2.40}$$

The specific acoustic impedance at place *x* in the tube is given by:

$$Z_s = \frac{p(x)}{u(x)} = -i \cdot \rho c \cdot \cot(k(l-x)) \tag{2.41}$$

As can be seen, the specific acoustic impedance in a closed tube does not equal the characteristic impedance but it is a co-tangent function that varies from plus to minus infinity and that in a standing wave tube particle velocity and sound pressure are 90 degrees out of phase.

The sound energy has a value in the tube but the sound intensity is zero. This makes sense because in the tube no sound is propagating: all the sound that is put in the tube is reflected. This follows also from the theory that is presented in the following paragraph.

As shown in Eq. (2.47) the time averaged sound intensity is defined as:

$$I = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} p(t)u(t)dt = \tfrac{1}{2}\hat{p}\hat{u}\cos(\varphi) \tag{2.42}$$

The phase shift in the tube is 90 degrees and therefore the time averaged sound intensity is zero.

In contrast to the (active) sound intensity, the reactive intensity is not zero. The reactive intensity is calculated as:

$$J = \tfrac{1}{2}\hat{p}\hat{u}\sin(\varphi) \tag{2.43}$$

As can be expected, if the phase shift is 90 degrees, the reactive intensity is large and the active intensity is low.

The reactive intensity alone is not of much relevance but when combined with the active intensity, it describes the sound field. A high value of the ratio of reactive to active intensity shows that we are in a near field where the pressure and the particle velocity are in quadrature. This knowledge is useful because sound measurements are affected by such conditions.

The ratio of reactive and active intensity is a very important figure to find out the quality of the pu intensity measurement.

Diffuse sound field

In a pure diffuse sound field sound is propagating in all directions with the same magnitude. Similar as in a standing wave tube, in a pure diffuse sound field the sound energy has a value and the intensity is zero: the sound energy is not propagating.

The difference between a standing wave and a diffuse sound field is that in a pure standing wave the phase shift is 90 degrees and in a diffuse sound field the phase shift is random. It takes all values, and when the position or

frequency is changed it will change in an unpredictable manner. In a diffuse field the sound pressure level and the particle velocity level do not vary in space, in a standing wave the sound pressure and particle velocity do vary much.

In a standing wave the reactive intensity is high and in a diffuse sound field the reactive intensity zero. In a diffuse sound field both active and reactive intensity are zero.

A diffuse sound field can be found in a reverberant room; a large room that has hard sound reflecting walls that are not parallel or perpendicular to each other.

Diffuse field; direct field and reverberation radius

In a reverberant room the sound pressure level is the same at all positions in the room, the same applies for the particle velocity level. One exception is at the position of the loudspeaker and at a certain radius around it. Here the sound levels are higher than the sound levels at other positions in the room. The point from where the sound level is not increased anymore is called the reverberation radius. Within the reverberation radius the sound field of the loudspeaker is dominant and outside the reverberation radius the diffuse sound field is higher than the direct field.

With a sound intensity probe only the direct field is measured, so the direct field can be determined outside the reverberation radius. This is because intensity of a diffuse sound field is zero.

Impulse like sounds

An impulse like sound source, i.e. a short event that occurs once, is difficult to interpret in the frequency domain. This is (a.o.) because the Fast Fourier Transform (FFT) works only for stationary signals. If a sound field is not stationary (so its statistical properties do vary with time) the FFT does not provide a meaningful answer.

If a single impulse like signal is generated, like a gunshot, it is possible to analyse the signal in the frequency domain. Signals like squeak and rattle noise in a car are short and distributed randomly in time and therefore impossible to analyse with FFT algorithms.

2.6 Representation of measurement results

Although signals are recorded as time signals, the representation of (stationary signals) in the frequency domain provides more insight for acoustic problems in most cases. The transformation from time domain to the frequency is called Fourier transform. FFT, Fast Fourier Transformation is a method that is used in computer programs to calculate the transformation. The terminology is explained here intuitively, without too much mathematics.

The Fast Fourier Transform

In a lot of cases sound fields are studied in the frequency domain. Normally the signals are captured in the time domain and transformed with the Fast Fourier Transform (FFT). The technique is not explained here in detail but in very brief steps it goes like this: the time signal is divided in multiple pieces of equal length.

Each part (or block) is windowed in a certain way (there are many ways). This windowing means that the block of time data is multiplied with a function that starts and ends with a zero value so that the beginning and the end of the block will be zero.

Each block is then transformed to the frequency domain. After the transformation of each windowed block the results are averaged.

Due to the nature of the FFT process (the signal is divided in pieces, windowed transformed and then averaged), the method only works for stationary signals.

Stationary signals are constant in their statistical parameters over time. If you look at a stationary signal for a few moments and then wait an hour and look at it again, it would look essentially the same, i.e. its overall level would be about the same and its amplitude distribution and standard deviation would be about the same. Rotating machinery generally produces stationary vibration signals.

So in short: acoustic and vibration phenomena are normally studied in the frequency domain. The FFT algorithm that is normally used dictates that the signals are stationary.

The first question in analysing signals is if the signal is stationary. If so it can be studied in the frequency domain.

An example of data that is represented in the time domain is a music CD. Music is stored on a cd and when it is put in a cd-player the data can be retrieved and listened to.

For measurement purposes one is often more interested in a representation in the frequency domain. If the time data is transformed in to the frequency domain one is able to find out how much sound was recorded in a certain bandwidth (frequency span). So if a music cd is transferred into the frequency domain one is able to see how much ‘noise’ was made at lower frequencies (if for example a contrabass was playing) or at high frequencies (if for example a violin or flute was playing).

Autospectrum Sxx

The autospectrum is the representation of a signal “x” in the frequency domain. It is often an average value, so it is possible to transform a signal from the time domain into the frequency domain but not vice versa.

It is possible to calculate the autospectrum of 5 minutes music. This results in a frequency representation (the frequency on the horizontal axis and the signal strength on the vertical axis) that represents the average

‘noise’ that was generated in the 5 minutes of music. Now one knows how much noise was generated at each frequency component but with this representation it impossible to ever listen to the music anymore.

A sine wave function in the time domain will result in a single point output in the frequency domain. As an example I whistled (with approximately 1kHz) in a microphone during a flight in an aeroplane. In the left of Fig. 2.11 the time representation is shown, and right the same signal is shown in the frequency domain.

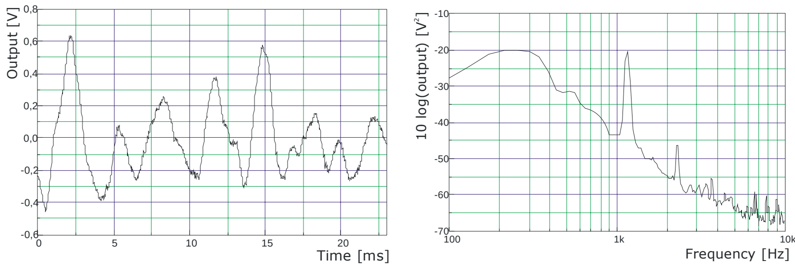

Fig. 2.11: Left: a time signal; Right the auto spectrum of that signal.

The mathematical equation behind this is given by the Fourier transformation:

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-ift} dt \tag{2.44}$$

When looking closer the Fourier transform can be understood. The integral is a time integrator and can be seen as a low pass filter in the frequency domain. So if the signal x is given by: $x(t) = \hat{x}\cos(\omega t + \varphi)$ the Fourier transformation works out as:

$$\begin{aligned} X(\omega_a) &= \int_{-\infty}^{\infty} \hat{x}\cos(\omega t + \varphi) e^{-i\omega_a t} dt \\ &= \int_{-\infty}^{\infty} \hat{x}\cos(\omega t + \varphi)\left[\cos(\omega_a t) + i\sin(\omega_a t)\right] dt \\ &= \frac{\hat{x}}{2}\int_{-\infty}^{\infty} \cos\left((\omega_a - \omega)t + \varphi\right) + \cos\left((\omega_a + \omega)t + \varphi\right) dt + \\ &\quad \frac{i\hat{x}}{2}\int_{-\infty}^{\infty} \sin\left((\omega_a - \omega)t + \varphi\right) + \sin\left((\omega_a + \omega)t + \varphi\right) dt \end{aligned} \tag{2.45}$$

Only if the center frequency ω_a equals the signal frequency ω, the cosine and the sine with the difference term ($\omega_a t-\omega t$) becomes time independent. All the time dependent cosine and sine functions are averaged to zero.

$$\begin{aligned} X(\omega_a) &= \int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt \\ &\underset{if\ \omega_a=\omega}{=} \frac{\hat{x}}{2}\int_{-\infty}^{\infty}\{\cos(\varphi)+\cos(2\omega t+\varphi)\}\,dt + \frac{i\hat{x}}{2}\int_{-\infty}^{\infty}\{\sin(\varphi)+\sin(2\omega t+\varphi)\}\,dt \\ &= \frac{\hat{x}}{2}\cos(\varphi)+\frac{i\hat{x}}{2}\sin(\varphi)=\frac{\hat{x}}{2}e^{i\varphi} \end{aligned} \quad (2.46)$$

So the Fourier transform is able to extract the frequency components of a given time signal.

The noise free cross spectrum Sxy

The cross spectrum is the frequency representation of what in the time domain would be related to the product of two signals "x" and "y".

To calculate a cross spectrum one needs two signals. Not only the amplitude of the signal is required but also the phase shift between the signals is necessary. The amplitude of a signal is often well understood but the phase is more difficult to comprehend.

If the example of the music-cd is taken again; one can calculate the cross spectrum of a stereo signal. If a singer is recorded with two microphones and the distance to the left microphone is a little less than the right one, the sound will be recorded with the left microphone a little earlier than with the right one. Due to this, the amplitude of both channels will be approximately of the same magnitude but due to the time difference a phase shift between the left and right signal will occur. A time shift in the time domain will produce a phase shift in the frequency domain.

The cross spectrum of a stereo recording is something that has no much meaning. A more meaningful use is the determination of the sound intensity.

The sound intensity is defined as the time averaged product of sound pressure, for example $p(t)=\hat{p}\cos(\omega t)$ and particle velocity (with a certain phase shift φ compared to the pressure signal) $u(t)=\hat{u}\cos(\omega t+\varphi)$:

$$\begin{aligned} I &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} p(t)u(t)dt = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\hat{p}\cos(\omega t)\hat{u}\cos(\omega t+\varphi)dt \\ &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\tfrac{1}{2}\hat{p}\hat{u}\left(\cos(\varphi)+\cos(2\omega t+\varphi)\right)dt \\ &= \tfrac{1}{2}\hat{p}\hat{u}\cos(\varphi)+\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\tfrac{1}{2}\hat{p}\hat{u}\cos(2\omega t+\varphi)dt = \tfrac{1}{2}\hat{p}\hat{u}\cos(\varphi)\equiv \mathrm{Re}\{S_{pu}\} \end{aligned} \quad (2.47)$$

As shown in Eq. (2.47) this time averaged intensity is a DC (non-time dependent) value. If both pressure as velocity have a certain frequency, the non-averaged output equals a DC value depending on the phase shift, plus a signal of double frequency. This double frequency part of the equation is averaged to zero.

If the signals are 100% correlated (so they have no noisy, uncorrelated components), the cross spectrum can be rewritten in the autospectra of both signals and the phase shift between them:

$$S_{xy}=\sqrt{S_{xx}}\sqrt{S_{yy}}(\cos\varphi+i\sin\varphi) \quad (2.48)$$

In the frequency domain the (active) intensity is defined as the real part of the cross spectrum of sound pressure and particle velocity. If the sound pressure and the particle velocity are free of noise, the cross spectrum can be expressed as the product of the autospectra:

$$I=\mathrm{Re}\left(S_{pu}\right)=\mathrm{Re}\left(\sqrt{S_{pp}}\sqrt{S_{uu}}(\cos\varphi_{pu}+i\sin\varphi_{pu})\right)=\sqrt{S_{pp}}\sqrt{S_{uu}}\cos\varphi_{pu} \quad (2.49)$$

If both pressure $p(t)=\hat{p}\cos(\omega t)$ as velocity $u(t)=\hat{u}\cos(\omega t+\varphi_{pu})$ have a certain frequency ω the cross spectrum is represented as a single value $\hat{p}\hat{u}$ at the (single) frequency ω and not at the double frequency.

The noisy cross spectrum Sxy

If the pressure $p(t)=\hat{p}\cos(\omega t)$ and velocity $u(t)=\hat{u}\cos(\omega t+\varphi)$ signals are not fully correlated, an additional noise component is to be taken into account. If two noisy signals from a pressure microphone and a velocity microphone are observed:

$$\begin{aligned} p(t) &= \hat{p}_s\cos(\omega t)+\hat{p}_n\cos(\omega t+\tilde{\theta}(t)) = P_S+P_N \\ u(t) &= \hat{u}_s\cos(\omega t)+\hat{u}_n\cos(\omega t+\tilde{\vartheta}(t)) = U_S+U_N \end{aligned} \quad (2.50)$$

The additional noise components P_N and U_N have constant amplitude and a phase shift $\tilde{\theta}$ that is varying randomly over time. If the intensity is calculated it will show that in theory the noise components do not affect the outcome:

$$\begin{aligned} I &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}p(t)u(t)dt=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}(P_S+P_N)(U_S+U_N)dt \\ &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}P_SU_S+P_SU_N+P_NU_S+P_NU_N\,dt \\ &= \tfrac{1}{2}\hat{p}_s\hat{u}_s\cos(\varphi)+\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}P_SU_N+P_NU_S+P_NU_N\,dt \\ &= \tfrac{1}{2}\hat{p}_s\hat{u}_s\cos(\varphi) \end{aligned} \quad (2.51)$$

The last simplification is possible because the time average product of two uncorrelated signals is zero, so e.g.:

$$\begin{aligned}\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}P_S U_N dt &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\hat{p}_s\cos(\omega t)\times\hat{u}_n\cos(\omega t+\tilde{\vartheta}(t))dt\\ &=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\tfrac{1}{2}\hat{p}\hat{u}\left(\cos\tilde{\vartheta}(t)+\cos(2\omega t+\tilde{\vartheta}(t))\right)dt\\ &=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\tfrac{1}{2}\hat{p}\hat{u}\cos\tilde{\vartheta}(t)dt=0\end{aligned} \quad (2.52)$$

This effect can be understood if one observes the phase: it is varying randomly in time so the product is half of the time positive and half of the time negative. Therefore the time average is zero. Of course the averaging time is not infinite and the noise in two signals is often not completely uncorrelated so the noise reduces not to zero. In practical cases the noise is reduced 10dB-40dB.

Conclusion is that in theory the noise component does not affect the cross spectrum of two signals. This is not the case for the auto spectrum as will be shown in the paragraph below.

The formal equations to calculate the cross spectrum are introduced via the correlation function which indicates the time average relationship between two signals in the time domain. The cross-correlation function between the sound pressure and particle velocity is given by:

$$R_{pu}(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_0^T p(t)u(t+\tau)dt \quad (2.53)$$

So, the sound intensity is given by:

$$I=\lim_{T\to\infty}\frac{1}{T}\int_0^T p(t)u(t)dt=R_{pu}(0) \quad (2.54)$$

The Fourier transform of the cross-correlation function gives the cross spectrum:

$$S_{pu}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}R_{pu}(\tau)e^{i\omega\tau}d\tau \quad (2.55)$$

Autospectrum versus cross spectrum

The auto spectrum of a signal is in fact nothing more than the cross spectrum of the signal with itself. Two identical signals are of course 100% correlated and the phase shift is zero, so Eq. (2.48) may be used:

$$S_{xy}\Big|_{x=y}=\sqrt{S_{xx}}\sqrt{S_{xx}}\left(\cos 0+i\sin 0\right)=S_{xx} \quad (2.56)$$

The uncorrelated parts of two noisy sources are reduced if the cross spectrum is used. When the autospectrum is taken from the noisy pressure signal from the previous example, the noise component does not reduce as can be seen in the expressions below:

$$\begin{aligned}S_{pp}&=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}(P_S+P_N)(P_S+P_N)dt=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}P_S^2+P_N^2+2P_SP_N dt\\ &=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}P_S^2+P_N^2dt=S_{P_SP_S}+S_{P_NP_N}=\tfrac{1}{2}\hat{p}_s^2+\tfrac{1}{2}\hat{p}_n^2\end{aligned} \quad (2.57)$$

The time averaged product of $2P_SP_N$ is again zero but the squared noise component P_N^2 is not. It shows that the noise is will not be reduced when the autospectrum is used.

If one wants to measure for example the autospectrum of the pressure that is generated by a very low level sound source, selfnoise of the transducer may be a problem. One can use two identical transducers and measure at the same location the cross spectrum of the two signals. The selfnoise will be reduced considerable that way. A reduction of 40dB can be achieved this way.

Transfer function H_{xy}

The transfer function of two signals is the frequency representation of what in the time domain would be the ratio of two signals "x" and "y".

Just like the cross spectrum, to calculate the transfer function one needs two signals. If the example of the music-cd is taken once more; one can calculate the transfer function of a stereo signal. If the same singer is recorded with the two microphones, just like the example before, the amplitude of both channels will be approximately the same but due to the time difference a phase shift of the stereo signal will occur. The phase shift that is calculated with the transfer function is the same as calculated with the cross spectrum but when the amplitude of both channels are approximately the same the magnitude of the transfer function will be approximately unity.

The transfer function is defined as the cross spectrum divided by the autospectrum of one of the signals.

$$H_{xy}=\frac{S_{xy}}{S_{xx}} \quad (2.58)$$

Coherence

The coherence is a value that is often used to check if measurements are valid. The value equals unity if two signals p and u 100% correlated. If so, the cross spectrum can be rewritten as the product of two autospectra:

$$coh = \frac{\left|S_{pu}\right|^2}{S_{pp}S_{uu}} = \frac{\left|\sqrt{S_{pp}}\sqrt{S_{pp}}\left(\cos(\varphi_{pu}) + i\sin(\varphi_{pu})\right)\right|^2}{S_{pp}S_{uu}} = \frac{\left|\sqrt{S_{pp}}\sqrt{S_{pp}}\right|^2}{S_{pp}S_{uu}} = 1 \qquad (2.59)$$

If the signals p and u are noisy they can be represented as done in Eq. (2.50). The cross spectrum will not be affected by the noisy component but both autospectra will be a larger value:

$$coh = \frac{\left|S_{pu}\right|^2}{S_{pp}S_{uu}} = \frac{\left|\frac{1}{2}\hat{p}_s\hat{u}_s\left(\cos(\varphi) + i\sin(\varphi)\right)\right|^2}{\left(\frac{1}{2}\hat{p}_s^2 + \frac{1}{2}\hat{p}_n^2\right)\left(\frac{1}{2}\hat{u}_s^2 + \frac{1}{2}\hat{u}_n^2\right)} = \frac{\hat{p}_s^2\hat{u}_s^2}{\left(\hat{p}_s^2 + \hat{p}_n^2\right)\left(\hat{u}_s^2 + \hat{u}_n^2\right)} < 1 \qquad (2.60)$$

An alternative method to calculate the Fourier response

The Fourier transformation can be done with an electronic network. The block diagram is shown in Fig. 2.12 [18].

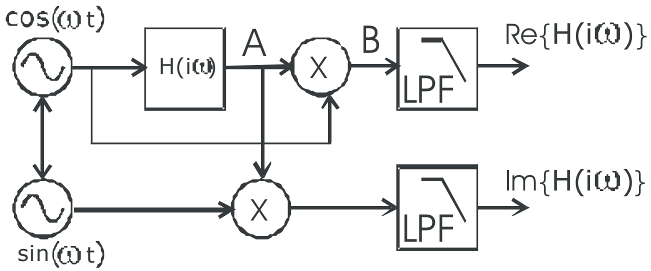

Fig. 2.12: Block diagram of a system that determines the imaginary and real part of a system by means of a sine signal.

If in Fig. 2.12 a sine wave cos(ωt) is submitted to a system H(iω), the signal at position 'A' of the system will be:

$$A = \cos(\omega t) \times \left|H(i\omega)\right|\cos(\omega t + \arg\{H(i\omega)\}) \qquad (2.61)$$

After multiplying signal 'A' with the input signal cos(ωt) the signal at position 'B' of the system is:

$$\begin{aligned} A &= \cos(\omega t) \times \left|H(i\omega)\right|\cos(\omega t + \arg\{H(i\omega)\}) \\ &= \left|H(i\omega)\right|\cos(\arg\{H(i\omega)\}) + \left|H(i\omega)\right|\cos(2\omega t + \arg\{H(i\omega)\}) \end{aligned} \qquad (2.62)$$

After proper low pass filtering:

$$A = \left|H(i\omega)\right|\cos(\arg\{H(i\omega)\}) = \mathrm{Re}\{H(i\omega)\} \qquad (2.63)$$

The imaginary part of the transfer function is determined in a similar manner.

If the input sine wave is swept through the frequency band of interest, the complex transfer function is known.

Time-frequency representation

A time-frequency representation (TFR) is a view of a signal (that is a function of time) represented over both time and frequency. A signal, as a function of time, may be considered as a representation with perfect temporal resolution. The magnitude of the Fourier transform (FT) of the signal may be considered as a representation with perfect spectral resolution but with no temporal information (although the magnitude of the FT conveys frequency content, it fails to convey where in time different events occur in the signal). The time-frequency representation provides some temporal information and some spectral information, simultaneously. They are used for the analysis of signals containing multiple time-varying frequencies.

As an example, a simulated signal which the time representation is depicted in Fig. 2.13 left. Clearly, this is an oscillating signal whose frequency varies with time. However, it is difficult to conclude from this representation the relationship between frequency vs. time. The spectrum of this signal does not give more indications, expect the fact that its frequency goes through the entire bandwidth see Fig. 2.13 right.

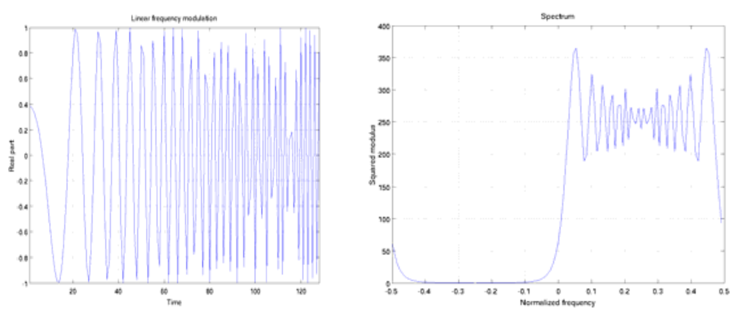

Fig. 2.13: Left: time representation of a signal; right: the frequency representation of the same signal.

The objective of time-frequency analysis is to offer a more informative description of the signal which reveals the temporal variation of its frequency contents. The plot in Fig. 2.14 gives an illustration: it shows that the signal is a simple sine sweep.

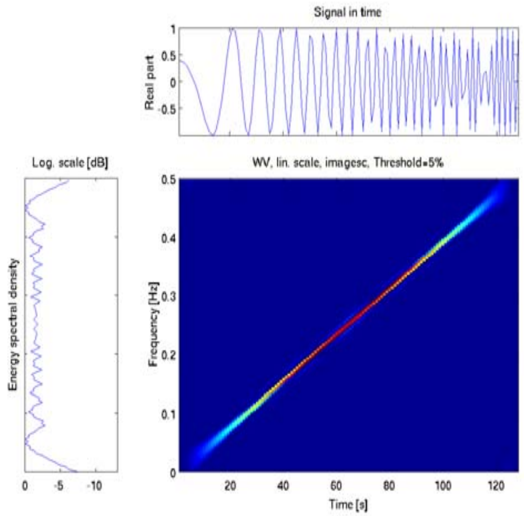

Fig. 2.14: time-frequency representation of a sine sweep signal.

2.7 Reciprocity

The following text is a summary from [1], [2], [3], [4] and [9].

In a linear elastic system, vibrational excitation at one point causes a response at another point. Generally speaking, if the system is passive and time-invariant, the transmission of vibration is invariant with respect to exchange of the points of excitation and observation. Such a system is reciprocal. In other words, reciprocity states that the transfer path in one direction equals the one in the opposite direction. The principle is valid in mechanical, electrical and acoustical systems as well as hybrid systems, e.g. acoustic transducers.

In its most elementary form the acoustic reciprocity principle states that an acoustic response remains the same when the source and receiver are interchanged.

Acoustic reciprocity was first mentioned by Helmholtz 1860 regarding sound transmission through pipes. A general theory of reciprocity, not limited to acoustics but valid for arbitrary systems containing friction and viscous damping is formulated in 1873 by Lord Rayleigh [10]. Lyamshev published a formal proof of these assumptions in 1959 [11]. He stated that any vibrating structure can be incorporated in the reciprocal system, thus paving the way for modern NVH applications.

Reciprocal measurements are found in NVH applications for the reason of the very different space requirements of sound sources and sensors; measurements of acoustic transfer functions are often much easier done reciprocally.

A typical example application is the measurement of acoustic transfer paths of sound radiated from a vehicle's engine to the driver's ear. The engine compartments of today's cars are almost completely filled with the engine itself and ifs various subsystems. Therefore a microphone can be installed much easier than a speaker, whereas in the cabin there should be enough space for a sound source.

Furthermore, reciprocal measurements can offer better results, and again size is a reason. Small sensors can be placed much closer to sound-radiating surfaces to which transfer functions are to be determined, and measurement positions can be chosen almost without restriction Thus the local sound field is sampled better than by the direct measurement, which employs a sound source inside the engine compartments that usually cannot be placed at the exact point of interest. Hence the reciprocal transfer function may approximate the real transfer path better than the direct measurement.

Moreover, when measuring reciprocally, e.g. with a sound source inside the cabin, all transfer paths are excited - and thus can be measured - simultaneously. This means a significant reduction in time requirements compared with the direct method, where each transfer function of interest has to be measured one after another.

Acoustic reciprocity

Loosely stated, the reciprocity principle states that the transfer function *p/Q* between a monopole sound source (*Q*) and the resulting sound pressure field (*p*) is unchanged if one interchanges the points where the monopole source is placed and where the sound pressure field is measured.

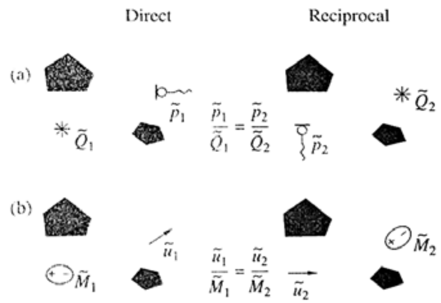

Fig. 2.15: Reciprocity for sound pressure with monopoles and particle velocity with dipoles, from [1].

In a similar way the relationship *u/M* between a dipole sound source (*M*) and the resulting particle velocity field (*u*) is unchanged if one interchanges the points where the dipole source is placed and where the particle velocity field is measured, see Fig. 2.15.

The latter can be deducted by the previous statement since a dipole sound source can be considered as two equal strength monopoles that are out of phase and a small distance, *d* apart (such that *kd*<<1) and particle velocity is linked to sound pressure gradient, see Eq. (2.10).

In the presence of impedance boundaries and if nonlinear sound propagation is avoided (sound pressure level less than about 135dB in air) the reciprocity principle is valid.

The reciprocity relation can be noted in a more general form by writing:

$$\iint_S u_2 \cdot n\, p_1 dS = \iint_S u_1 \cdot n\, p_2 dS \tag{2.64}$$

p_1 and u_1 and p_2 and u_2 are two arbitrary sound fields in two arbitrary points in a certain volume V that is enclosed by the surface S and *n* is the is a surface normal vector pointing into the volume V.

So p_1 is the sound pressure in point one in sound field one and u_1 is the particle velocity in point two in sound field one. Subsequently p_2 is the sound pressure in point one in sound field two and u_2 is the particle velocity in point two in sound field two.

In chapter 10: 'source path contribution', some applications of the reciprocity principle are shown.

Vibro-acoustic reciprocity

The transfer function between a mechanical vibrational force applied to an elastic plate or shell and the resulting sound pressure (*p/F*) equals the ratio *v/Q*. Where *v* is the particle velocity determined close to the surface and *Q* is a monopole sound source, see Fig. 2.16: *p/F=v/Q*.

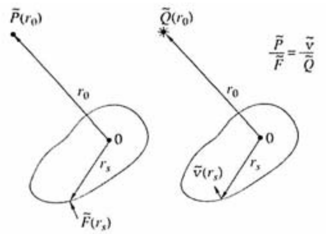

Fig. 2.16: Reciprocity of an applied force on a surface and sound pressure with particle velocity at the surface and a monopole sound source.

2.8 Point sources

The simplest source to describe mathematically is a pulsating sphere with radius *a*, that expands and contracts harmonically with spherical symmetry. If the radius of the sphere is much smaller than the wavelength (*ka*<<1), the source becomes a monopole or in other words a point source. Such sources are difficult to create in practice but the monopole sound source is a central concept in theoretical acoustics. Also for reciprocal measurements the point source is an essential tool.

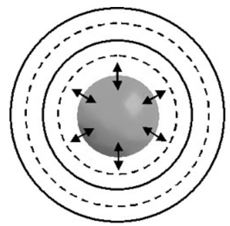

Fig. 2.17: A pulsating sphere can be regarded as a monopole if the radius of the sphere is small compared to the wavelength.

The sound pressure generated by the point source is given by:

$$p = i\frac{\rho\omega Q}{4\pi r} e^{i(\omega t - kr)} \tag{2.65}$$

With *r* the distance to the source and *Q* the source strength [m^3s^{-1}]. The sound power is given by:

$$P = \frac{\rho c k^2 \left|Q^2\right|}{8\pi} \tag{2.66}$$

It is proportional to the square of the frequency, indicating that a small pulsating sphere is not an efficient radiator at low frequencies. The acoustic impedance of a point source is given in Eq. (2.25). With that, the particle velocity field can be calculated.

$$\begin{aligned} p(r,t) &= i\rho ck\frac{Q}{4\pi r}e^{i(\omega t - kr)} \quad , \quad Z = \frac{p}{u} = \rho c\frac{ikr}{ikr+1} \\ u(r,t) &= \frac{Q}{4\pi}\frac{ikr+1}{r^2}e^{i(\omega t - kr)} \end{aligned} \tag{2.67}$$

As can be seen, relative close the source (*kr*<1), the particle velocity field is decreasing with the square of the distance and in the far field (*kr*>1), the particle velocity field is decreasing proportional with distance.

Practical realisations of point sources are found in chapter 9: ‘monopole sound sources’.

2.9 The Helmholtz Integral Equation

There is much work in many fields of acoustics which relies heavily on the Helmholtz Integral Equation. In particular, the field of boundary elements has as its starting point this important equation (paragraph is summary from [8] and [12]).

Fig. 2.18 illustrates a sound radiating object with an arbitrarily shaped, but closed, boundary surface *S*.

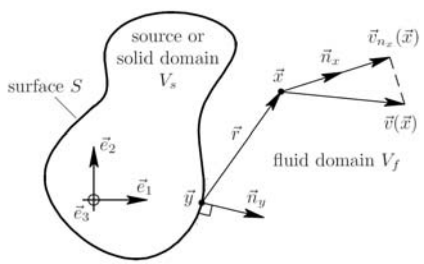

Fig. 2.18:Nomenclature in the exterior acoustic problem.

An integral equation is known as the Helmholtz integral equation. It relates the acoustic pressure and normal velocity on the closed boundary surface *S* of a vibrating object to the radiated pressure field in the fluid domain:

$$p(\vec{x}) = \oint_S \left\{ \frac{\partial G(r)}{\partial n_y} p(\vec{y}) + i\omega\rho G(r) v_{n_y}(\vec{y}) \right\} dS + p_{in}(\vec{x}) \qquad (2.68)$$

In a field point $\vec{x}$, the $p(\vec{x})$ can be calculated. The point $\vec{x}$ is also called the evaluation point because the integral has to be evaluated for each point $\vec{x}$. The unit normal to the surface at source point $\vec{y}$, denoted as n_y, is pointed into the fluid domain. Distance *r* is the length of vector $\vec{r}$ that is directed from the source point $\vec{y}$ to the field point $\vec{x}$: $r = \|\vec{x} - \vec{y}\|$. The term p_{in} represents the incident acoustic wave in the case of a scattering analysis. In an unbounded fluid domain without reflecting objects (free space), Green’s function *G(r)*, also referred to as the kernel of the integral equation, is:

$$G(r) = \frac{e^{-ikr}}{4\pi r} \qquad (2.69)$$

Physically, *G(r)* represents the effect observed at point $\vec{x}$ created by a unit source located at point $\vec{y}$. However only one of the unknown variables has to be prescribed on the boundary *S* to solve the problem. In engineering situations, the usual given surface normal velocity on the surface *S* is used.

It can be proved (see e.g. chapter 8, [12]) that an alternative Green’s can be found so that:

$$\frac{\partial G_N}{\partial n_y} = 0$$

This is called the Neumann Green function. Now the Helmholz equation (Eq. (2.68)) alters in:

$$p(\vec{x}) = \iint_s i\omega\rho G_N v_n dS \qquad (2.70)$$

It can be understood that the Neumann Green function can be found experimentally by e.g. a reciprocal method. With such measurement the sound pressure in a field point $p(\vec{x})$ is related to a normal particle velocity at a surface.

The previous relation can be rewritten in the discrete form if the surface *S* divided in small sub surfaces ∆S so that in each sub surface the normal velocity and the Neumann Green function can be considered constant:

$$p(\vec{x}) = i\omega\rho \sum G_N v_n \Delta S \qquad (2.71)$$

This relation is used in the source path contribution method that is explained in chapter 10. The Neumann Green function is measured in a reciprocal way. With that Neumann Green function the pressure contribution of each part in e.g. a car is determined by measuring the surface particle velocity.

2.10 Sound & vibration sensors

In this chapter the most common sensors that detect sound and vibration are presented.

Transducers are devices that convert one quantity into another. Membrane microphones are acoustic transducers that are commonly applied to convert sound pressure into electrical signals. The sound pressure “pushes” a membrane forward and backwards and the deflection of the membrane is detected electrically. When this membrane closes a cavity the microphone detects sound pressure. In a construction in which both sides of the membrane are subject to the acoustic wave, the microphone is sensitive for sound pressure gradient.

Sound, however, consist of two elements: sound pressure and particle velocity. A pressure microphone is constructed to measure sound pressure whereas the Microflown enables the direct and reliable measurement of particle velocity.

To determine the amount of noise, sound intensity has to be measured. Two sensors are required for the determination of sound intensity. The most obvious way is measuring sound pressure (p) with a microphone and particle velocity (u) with a Microflown, and then multiply and time integrate both signals. This is called the p-u method. Different methods use closely spaced microphones (p-p method) or Microflowns (u-u method).

Measuring sound pressure

The detection of sound pressure of a microphone is similar to the way the human ear perceives sound. A membrane closes of a cavity, inside the cavity the pressure equals the static air-pressure and pressure variations outside the cavity cause a movement of the membrane. This movement is detected. In this paragraph the functioning of a capacitive microphone is explained.

All practical microphones detect the (sound pressure induced) deflection, or movement, of a membrane. There are several ways to implement this.

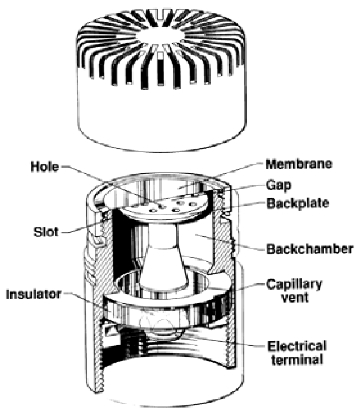

Fig. 2.19: A cross-section of a condenser microphone.

Electromagnetic microphones consist of a coil connected to the membrane. The movement of the membrane causes the coil to move in a magnetic field resulting in an electrical signal. The first who developed this type of microphone was the German teacher Phillipp Reis [15] in the year 1860. Since then the development of sound pressure sensors began.

The capacitive microphone detects the acoustic pressure variations by means of a condenser [16]. The membrane acts as one plate of the capacitor whereas a perforated-rigid-back-electrode is the other plate. A

deflection of the membrane causes a variation of the capacitance. This type of microphone needs an additional (bias) voltage for operation and is called the condenser microphone. To avoid external biasing it is possible to provide one of the plates with an electret, an electrically charged layer, creating a so-called electret microphone. Capacitive microphones have good noise properties (or signal to noise ratio), which is the most important parameter of a microphone. The first condenser (electret) microphone was built in 1918 [16]. The construction of today's measurement condenser microphone is illustrated in Fig. 2.19.

Although several different ways to detect deflection of a membrane exist, the previous mentioned are the most common. To get a better understanding of the sound pressure measurement, the condenser microphone is investigated further.

Two electrical conducting plates separated by an insulator will create a condenser. The condenser microphone consists on an electrical conducting, stretched membrane over an electrical conducting backplate forming together, with the air gap (insulator), a condenser. Sound pressure excites the membrane, changing the capacitance of the microphone. This capacitance is electrically measured. The performance of a condenser microphone depends upon the design of its electrical and mechanical systems.

A high polarisation voltage, large membrane radius, small air gap, and low membrane tension favour the sensitivity. Sensitivity is a microphone's electrical output caused by a sound pressure given in Volts per Pascal.

The bandwidth, the range of frequencies within which a microphone yields a usable output signal, is favoured by a high membrane tension, small membrane radius and low surface mass density (implying a thin membrane). The response of a microphone "rolls-off" at low frequencies (frequencies lower than 1 to 125Hz) due to the presence of a capillary vent, see Fig. 2.19. The purpose of this vent is to provide a path for pressure equalisation on both sides of the membrane following a change in ambient pressure. However, this vent supplies also a path for low-frequency acoustical waves to reach the backside of the membrane and thus suppresses the low-frequency response of the microphone.

The selfnoise is the electrical output of a microphone (in an absolute silent environment) caused by noise, expressed in a quantity corresponding to a certain sound level. Resistive damping of the membrane by the air layer and the holes in the backplate (see Fig. 2.19) in the air gap causes the thermal noise of a condenser microphone. For lower frequencies, the preamplifier 1/f-noise dominates. The thermal noise of the microphone is an inverse function of the membrane radius. If the sensitivity of the microphone is low the preamplifier noise becomes more dominant.

The preamplifier is needed to convert the very high impedance of the capacitive microphone to a lower output impedance. The voltage gain of these preamplifiers is usually unity.

The non-linear relationship between the capacitance and the membrane displacement is the main reason for the harmonic distortion at high sound pressure levels. A high membrane tension and small membrane radius will increase the upper sound pressure limit.

A polarisation voltage causes an attracting force between the membrane and the backplate. If the polarisation voltage is chosen too high, the membrane will collapse to the backplate.

A polar pattern (or microphone directivity) expresses the sensitivity to the angle of incidence of the sound field. A sound pressure microphone should be omni-directional which implies a constant sensitivity for any angle of incidence of the sound field. The larger the membrane radius (and thus physical size) the more the microphone behaves as an obstacle in the sound field. Due to this effect the directivity of the microphone increases for higher frequencies. Another cause of directivity of sound pressure microphones is related to the diameter. When the wavelength is becoming smaller than the diameter, and the angle of incidence sound wave is perpendicular to the membrane, one part of the membrane is pulled and another part is pushed. The sensitivity will be reduced due to this phenomenon and due to this the sound pressure microphone becomes directional.

A microphone with a half-inch membrane radius and 200V polarisation voltage is chosen for general measuring purposes. This is because the microphone has a relatively low selfnoise (20dB(A)), a sufficiently large bandwidth (20kHz) and reasonable omni-directional properties (3dB variation at 8kHz). The microphone can be used for sound levels up to 160dB.

The half inch pressure microphone is used most commonly. The bandwidth of such microphone is specified as higher than 20kHz but the frequency response is not flat for frequencies higher than 4kHz.

For higher frequencies the frequency response becomes directional. Therefore two types of ½” microphones are designed, a pressure microphone and a free field microphone.

Table 2.2: General microphone specifications related to the membrane radius

Membrane radius	Sensitivity [mV/Pa]	Bandwidth [kHz]	Selfnoise dB(A)	Upper limit [3% distortion]
1"	50	10	10	150 dB SPL
½"	12,5	24	22	160 dB SPL
¼"	4	100	36	170 dB SPL
1/8"	1	160	55	180 dB SPL
1/10" (Knowles)	30	20	27	100 dB SPL

Fig. 2.20 shows how mechanical properties affect the frequency response and dynamic range and table 2.2 shows how dynamic range, bandwidth and sensitivity depend on microphone size. When the membrane diameter decreases, the maximum sound pressure level a microphone can withstand, and the noise floor will increase. Sensitivity is proportional to the diaphragm area. In actual microphone design, other parameters may be changed along

with the membrane radius so that particularly useful compromises in bandwidth, dynamic range, and sensitivity can be achieved. The upper four microphones are used for measurement purposes and the Knowles is used for audio pick-up.

The cylindrical microphone (Knowles Electronics) for instance has a diameter of 0.1" (the company Microtronic has a similar microphone). This microphone is used for hearing aid purposes. The selfnoise of this miniature microphone is 27dB(A), the effective* bandwidth however is limited to approximately 20kHz (1/8” B&K has a bandwidth of 160kHz), and the upper limit of the sound pressure is 100dB SPL. The sensitivity is 30mV/Pa. The high sensitivity and low sound-pressure upper-limit suggest low membrane tension.

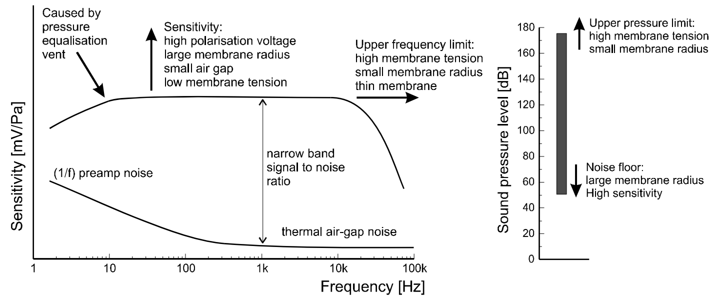

Fig. 2.20: The effect of mechanical properties on acoustical specs of condenser microphones.

The lower membrane tension will result in a different mechanical design i.e. air layer and holes in the backplate; therefore the selfnoise due to the resistive damping of the membrane by the air layer and the holes in the backplate is lower than the 1/8” B&K. Due to the low membrane tension the sensitivity is high and the preamplifier noise becomes less dominant.

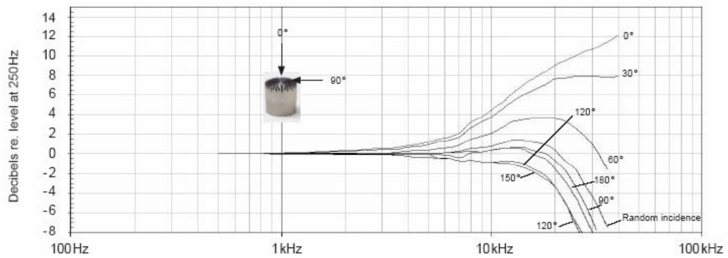

Fig. 2.21: free field corrections for various angles of incidence.

A Free-field microphone is designed to measure the sound pressure in a sound field and compensate for the influence of its presence in the sound field. A free-field microphone measures the sound pressure as it existed before the microphone was introduced into the sound field; in effect, maintaining free-field conditions. A free-field microphone should be pointed towards the sound source, i.e. at an angle of 0º incidence to the source.

A pressure microphone is designed to measure the sound pressure that actually exists in front of its diaphragm. This means that the microphone also measures any changes in the sound field brought about by the presence of the microphone. A pressure microphone is used, typically, in coupler measurements and, when flush-mounted on a wall for example, for sound-pressure measurements on the surface of the wall. In such applications, the effect of the microphone's presence in the sound field is intended.

The difference in the two designs is the different frequency response.

As can be seen in Fig. 2.21, at 20kHz the frequency response of a ½” microphone varies 13dB as function of angle of incidence. Sound pressure is a scalar value and a sound pressure microphone should not be directional. It is therefore in general recommendable to use a smaller microphone if sound pressure is measured above 4kHz.

Measuring sound pressure with a Microflown sensor

There are various ways to be able to measure sound pressure with a (particle velocity sensitive) Microflown. If a Microflown is positioned very close behind a membrane the vibrations of the membrane will be sensed. The membrane is pressure sensitive so the Microflown will be too. In this case the sensing part of a pressure microphone is replaced by a Microflown. (See also chapter 7: “vibration measurements”).

It is also possible to put a Microflown in a standing wave tube. If the sound pressure in increased in front of the tube air will flow in to the tube and if the sound pressure is reduced, air will flow out. So the tube transforms sound pressure into particle velocity.

In stead of a tube it is also possible to use a Helmholtz resonator and put the Microflown in the throat. Again, sound pressure causes air to flow in and out of the resonator.

The microphones that are made by this techniques are not better, see further chapter 13: “the pressure Microflown”.

The Human ear

The human ear is sensitive for sound pressure. Hearing is a subjective response to sound pressure. At very low frequencies or very high pressure-levels, additional sensations are experienced but normally the ear is the acoustic receiver. It receives the vibrations on the eardrum, multiplies them by means of small bones in the middle ear, and transmits the vibrations through a fluid to nerve endings within the inner ear. These nerves transmit

an impulse to the brain that analyses and translates the acoustic impulse into a concept or situation.

In other words the essential difference between the detection of sound with a pressure microphone and hearing is that hearing is the combination of two pressure sensors and a processing unit. A microphone can only detect the sound pressure, interpretation must be done afterwards.

Loudness is the physical response to sound. At any given frequency between 20Hz and 20kHz the loudness varies directly with the sound pressure, but not in a simple straight-line manner. In Fig. 2.22 the equal loudness contours are shown, they represent the sound pressure level necessary at each frequency to produce the same loudness response in the average listener. As can be seen loudness is a very non-linear physical response. Humans are quite deaf at low frequencies. At 20Hz the level must be almost 70dB higher (ten million times as much sound energy) as at 2kHz to produce hearing. Another non-linearity appears at low frequencies: if, for example, the sound level is rising from 80dB to 90dB, humans perceive a 20dB increase of the sound pressure.

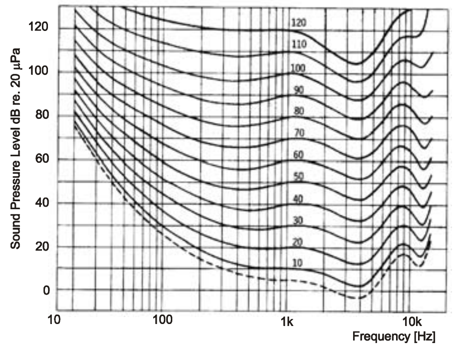

Fig. 2.22: Equal loudness contours of the human hearing.

To convert sound measurements performed with a microphone in one figure representing loudness, the “A”-weighing curve is introduced:

$$|W_A(f)| = 1.26 \frac{(12200\, f^2)^2}{(f^2+20.6^2)\,(f^2+12200^2)\,\sqrt{(f^2+107.7^2)}\,\sqrt{(f^2+737.9^2)}} \qquad (2.72)$$

This “A”-weighing curve is similar to the inverse of the equal loudness contour of 0dB, as shown in Fig. 2.22.

Measuring sound pressure gradient

The difference between the configuration of a sound pressure microphone and a (sound pressure) gradient microphone is that the gradient microphone has two (in stead of one) acoustic entrances separated with a distance L, see Fig. 2.23A. The membrane will deflect due to a sound pressure difference over the membrane. (In fact a pressure microphone detects a pressure gradient over the membrane as well, but in the back chamber the pressure is constant).

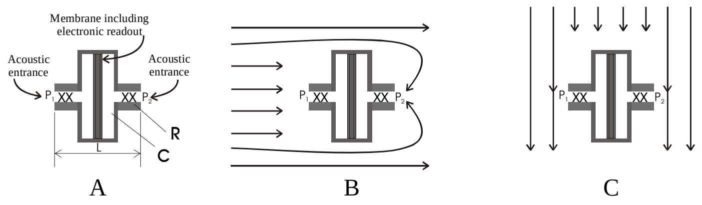

Fig. 2.23: A cross-section of a pressure gradient microphone.

If a sound wave enters in the sensitive direction, see Fig. 2.23B, the sound wave reaches first input P1 and a while later input P2. In this way the pressure gradient is determined; the output of this type of microphone will be linear with respect to: *p(x)-p(x+*Δ*x)*. Observing Eq. (2.10), it is not surprising that this type of microphone is also called "velocity microphone" since the particle velocity can be calculated by a pressure difference of two closely spaced microphones.

The velocity microphone will not be sensitive in the direction perpendicular to the sensitive direction, see Fig. 2.23C. The sound wave reaches both sides of the membrane at the same time and therefore the subtraction *p(x)-p(x+*Δ*x)* is zero.

It can be deduced that the sensitivity as function of the angle entry (the polar pattern) is a cosine function, the response is plotted in Fig. 2.24. At the right side this cosine function is shown. This shape of polar pattern is often called a "figure of eight". Normally a logarithm representation is preferred; it is shown at the left side.

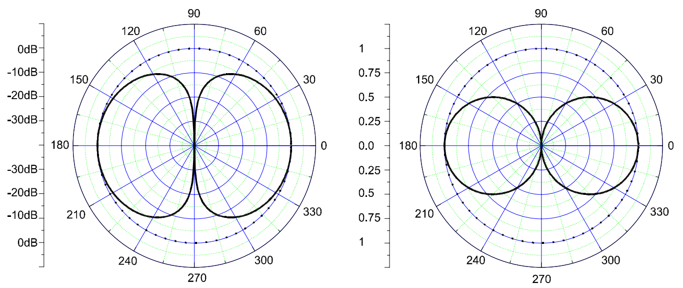

Fig. 2.24: Response of a sound pressure microphone (dashed line; omni directional) and a sound pressure gradient microphone (solid line; figure of eight).

For harmonic sound waves (*p(t,x)=p0 cos(*ω*t-*ω*x/c)*), the frequency response of a gradient microphone is not constant; the response increases linear with frequency:

$$p(x,t) - p(x+\Delta x,t) = \frac{p(x,t) - p(x+\Delta x,t)}{\Delta x}\Delta x \approx \frac{d}{dx}p(x,t)\Delta x = \frac{\omega}{c}p(x,t)\Delta x \qquad (2.25)$$

When the frequency increases so much that such that half the wavelength equals the separation distance, the sensitivity becomes maximal: two times higher than if the membrane was used in one-input configuration (regular pressure microphone). After this maximum the sensitivity drops dramatically, see Fig. 2.25. To show this effect, the frequency response due to a separation distance of 5cm is plotted: as can be seen the sensitivity is at its maximum at 3.3kHz; (½λ=½c/f=0.5x330/3300=5cm).

To create a useful output signal the response should be integrated (flat frequency response). Simply said, in the frequency domain integrating is the same as multiplying the sensitivity with 1/f.

Integrating can quite easily be achieved with an electronic circuit. In this case, for decreasing frequencies the signal to noise ratio will decrease.

It is also possible to use acoustic elements to achieve the integrating. Very simply explained, a dissipating element R is placed between the acoustic inputs and the membrane (that is stretched very loose), indicated with "xxx" in Fig. 2.25 and a large acoustic chamber creating a compliance C are together creating an acoustic RC low pass filter (integrator). The resulting frequency response will be reasonable flat from 250Hz up to 16kHz. Due to the lesser damping of the pressure gradients' membrane, the inherent noise of the capsule is much lower than a sound pressure sensitive microphone. However, the frequency response is filtered by the acoustic low-pass filter and therefore the selfnoise of a pressure gradient microphone is a few dB worse than a pressure microphone. For audio applications where signal to noise values should be as high as possible, acoustic integration is used whereas for measurement purposes where linearity and a flat frequency response is the most important issue, electronic integration is employed.

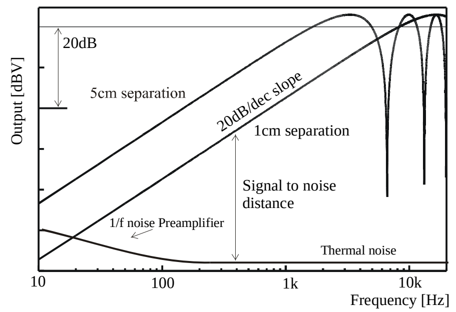

Fig. 2.25: Response of a pressure gradient microphone with an acoustic input separation of 5cm and 1cm.

Integration of the output of a 5cm-separation configuration results in a flat frequency response from low frequencies up to 3.3kHz. For frequencies higher than 3.4kHz the response will be useless.

To achieve a useful output signal for high frequencies the separation distance should not exceed 1cm. But as can be seen in see Fig. 2.25 for low frequencies the signal to noise ratio is very poor compared to a regular pressure microphone.

As stated, the output signal is integrated to create a flat frequency response. When corrected electronically afterwards, the noise is also integrated and the signal to noise ratio therefore is not affected by this correction. The selfnoise of a sound pressure gradient microphone (pair) that makes use of electrical correction afterwards is therefore always worse than a sound pressure microphone. When corrected acoustically beforehand, the frequency response will lack the first decade (20Hz-250Hz) and the noise figures will be only a few decibels worse than sound pressure microphones. A good sound pressure gradient microphone (Schoeps MK8) has a selfnoise of 19dB(A) and a good sound pressure microphone (Schoeps MK2H) has a selfnoise of 13dB(A) to 15dB(A).

Measuring particle velocity

The direct measurement of particle velocity is quite new. Until the invention of the Microflown in 1994, the particle velocity could only be measured indirectly. Here three velocity detecting methods will be described: the ribbon microphone, an ultrasonic detection method and the Microflown.

As stated before, sound pressure gradient shows a great resemblance to the particle velocity. After all, Eq. (2.10) states that the pressure gradient is proportional to the particle velocity. However in Fig. 2.25 one can see that the sensitivity of a pressure gradient microphone increases proportional with the frequency. For zero Hertz (a DC or constant particle flow) the sensitivity is zero. The Microflown however originates from a mass flow

sensor; a sensor that is designed to measure DC-flow. The fast response time and improved sensitivity is the only difference between a mass-flow sensor and a Microflown.

A general behaviour of pressure gradient microphones is the lower sensitivity for lower frequencies and ‘comb filtering effect’ at high frequencies.

Unlike this crucial difference between pressure gradient microphones and Microflowns there are also similarities. For instance the polar pattern is the same for a Microflown and a pressure gradient microphone. The near field effect (see above “the one-dimensional wave equation”) is also noticed in the same manner.

Another type of “velocity” microphone is the so-called **ribbon microphone** [17]. The operation principle of this type of microphone is again based on pressure gradient. In this case an aluminium foil (the ribbon) is placed under a low tension in a strong magnetic field, see Fig. 2.26. The difference in sound pressure between the two sides drives the ribbon and the velocity of the ribbon has the same response as the pressure gradient microphone. The output voltage due to the motion of the ribbon however is proportional to the product of the flux density, the length and the velocity of the ribbon. The result is a “flat” frequency response. (The ribbon moving in a magnetic field has an integrating character, whereas the pressure gradient microphone integrating the integration of the signal have to be performed electronically afterwards or acoustically on forehand).

The ribbon velocity microphone is a pressure gradient type of microphone that has (thus) no DC sensitivity. The selfnoise of such microphones is quite low since there is (of course) no thermal noise of the airgap. The main noise source is thermal noise of the electrical resistance of the ribbon.

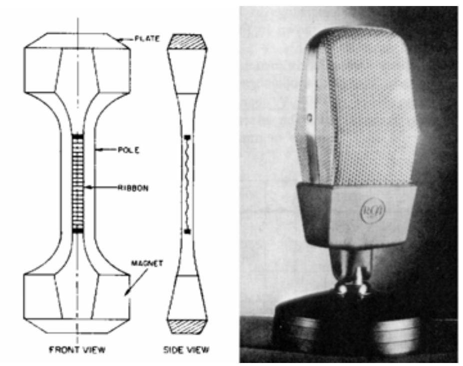

Fig. 2.26: A ribbon type of microphone. Left: schematic overview; right: the RCA BK11A velocity microphone [17].

A complete other way of sensing particle velocity is the principle of **ultrasonic transduction**: two parallel ultrasonic beams are launched in opposite directions. The travelling time from the transmitter to the receiver is linear proportional to the speed of sound in the air. When the air is moving this movement should be added to this speed. The probe consists of two transmitter-receiver pairs that are positioned opposite directions, see Fig. 2.27. The difference signal of the ultrasonic sound waves is proportional to the particle velocity; this type of velocity probe is also capable to measure DC flow. It is a true particle velocity sensor but it is a distributed sensor: it doesn't determine the particle velocity in one spot.

This probe however is not used very often since it is very sensitive for DC flows as for example wind or movements of the probe. Furthermore it is, just like the p-p probe, a distributed sensor with the problems associated with this type of sensing (for example maximal frequency limited by the spacing). Reflections of the ultrasonic signals caused problems when the probe was used nearby reflecting objects and at last, due to it physical dimensions, the probe is difficult to calibrate.

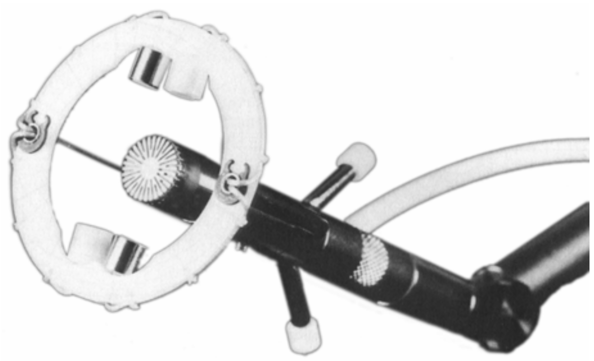

Fig. 2.27: Type 216 p-u intensity probe (Norwegian electronics).

If the signal from only one ultrasonic beam is observed one notices quite large a pressure sensitivity. This is caused by the temperature variations that are generated by the pressure variations of the sound field. The temperature variations cause a variation in the speed of sound which is measured to detect particle velocity with this method. To get rid of the unwanted pressure sensitivity a second ultrasonic beam is used in the opposite direction. The pressure sensitivity of this second beam is the same as the first one but the particle velocity sensitivity has a phase shift of 180 degrees. The subtraction of both signals got rid of the pressure sensitivity.

A (stereoscopic) **Laser Doppler Velocimeter** is measurement device used for estimating particle velocities, by means of specific signal processing of scattered light. It is an optical technique allowing direct measurement of local and instantaneous particle velocity. Its principle is based on the determination of the Doppler shift of light scattered from

seeding particles (tracers) suspended in the fluid. Two laser beams of equal intensity are focused and crossed at the point under investigation, forming an ellipsoidal volume consisting of equidistant dark and bright fringes. The scattered light is collected on a photomultiplier.

The Microflown is a particle velocity sensor that is based on a thermal principle. It consists of two very closely spaced and thin wires of silicon nitride with an electrically conducting platinum pattern on top of them. An example of an older type of Microflown is shown in Fig. 2.28. The sensor is made from silicon bulk material with platinum electrical connections on top of it and two platinum temperature sensors. At the top of the die one can see two sensors sticking out. The electrical connecting wirebonds are also visible.

The size of the two wires is 1mm in length, 5μm in width and 200nm in thickness. The metal pattern is used as temperature sensor and heater. The silicon nitride layer is used as a mechanical carrier for the platinum resistor patterns. The sensors are powered by an electrical current, causing the sensors to heat up. The temperature difference of the two cantilevers is linear dependent on the particle velocity level.

The operation principle will be explained briefly here, a more detailed mathematical model of the Microflown model is presented in chapter 3. The two squares S1 and S2 in Fig. 2.29 represent the two temperature sensors of the Microflown. The temperature sensors are implemented as platinum resistors and are powered by an electrical current dissipating an electrical power, causing them to heat up. An increase of the temperature of the sensors leads to an increase of the resistance as well.

The sensors have a typical operational temperature of about 200ºC to 400ºC if no particle velocity is present and all the heat is transferred in the surrounding air. When particle velocity is present, it alters the temperature distribution around the resistors. The temperature difference of the two sensors quantifies the particle velocity.

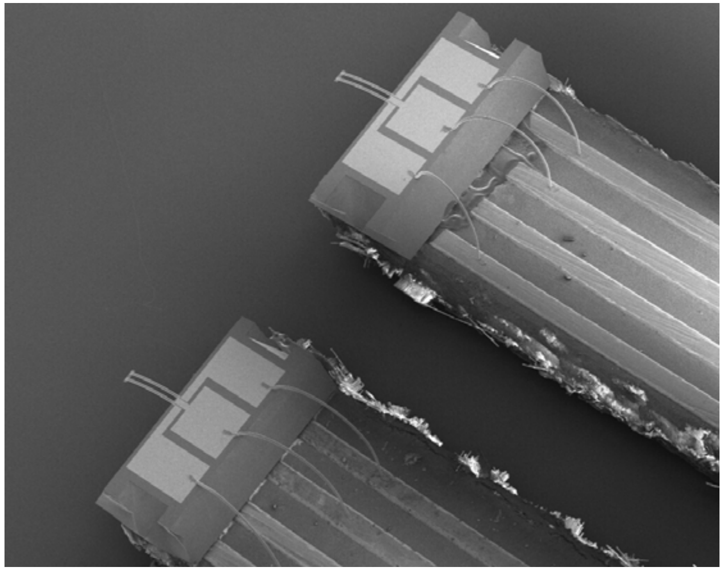

Fig. 2.28: Scanning electron microscope photo of cantilever type of Microflowns. The two wires that are sticking out are the actual Microflown, the three squares are the "bondpads" that are used to make electrical connections and the three wires in the lower part of the picture are wirebonds: Aluminium wires that are 80 micron in diameter. See further chapter 1 and 3.

Particle velocity causes a convective heat transfer of both sensors which will create a temperature drop of both sensors. The upstream sensor however, drops more in temperature than the downstream sensor since the downstream sensor is heated by the upstream convective heat loss see Fig. 2.29. A temperature difference is the result. This temperature difference is proportional to the particle velocity.

Not all the convective heat loss of the upstream sensor will be transferred to the downstream sensor; a certain percentage will be lost. This percentage will rise if the sensors are positioned further apart from each other. If, on the other hand, the sensors are brought closer together another phenomenon will become dominant. The particle velocity induced temperature difference will cause a conductive heat flow in the opposite direction. This feedback heat flow will temper the sensitivity. The closer the sensors are placed, the more conductive heat flow will take its effect.

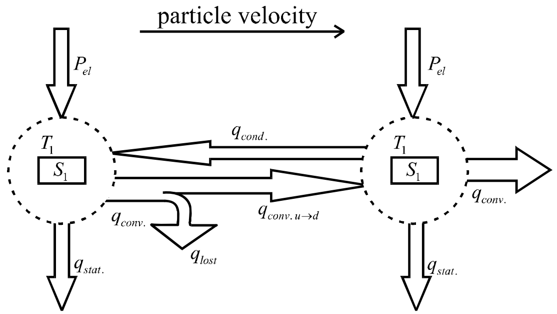

Fig. 2.29: Schematic overview of the heat flows around a Microflown.

At higher frequencies the sensitivity of the Microflown is decreasing. This high frequency roll-off that is caused by the time it takes heat to travel from one wire to the other can be estimated by a first order low pass frequency response that has a (diffusion) corner frequency in the order of 1kHz. The second high frequency roll-off is caused by the heat capacity (or thermal mass) and shows an exact first order low pass behaviour that has a heat capacity corner frequency in the order of 10kHz for modern Microflowns.

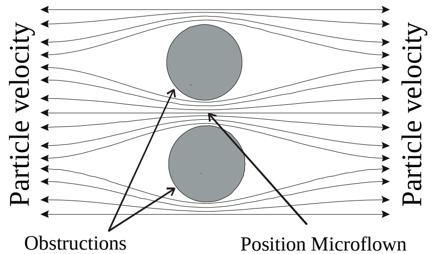

Fig. 2.30: A well-chosen package will result in a particle velocity gain (top view).

To protect its fragile sensors, the Microflown has to be packaged. An positive aspect of the packaging is that the particle velocity level increases when a well-chosen obstacle is placed near the sensors, see Fig. 2.30 and Fig. 2.31. The so-called package gain is caused due to a "channelling" of the particle velocity, this can result in an increase of the particle velocity level of a factor 3 up to 30 times (+10dB to +30dB).

Since the noise has a thermal electrical origin, it is not be affected by the package (gain). The selfnoise of a packaged Microflown therefore can be expected 10dB to 20dB less than a non-packaged Microflown.

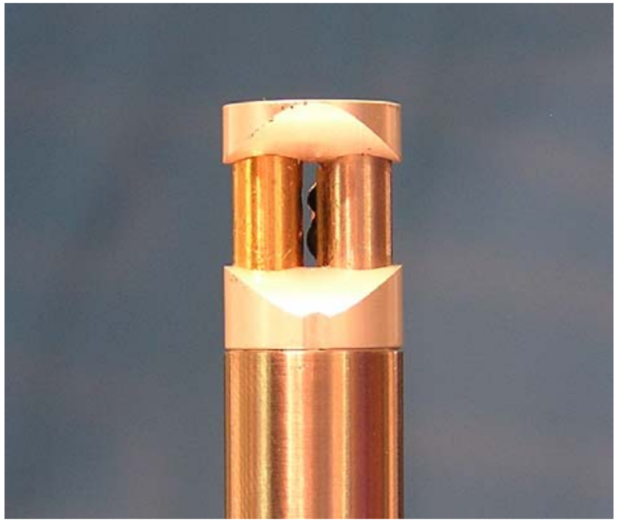

Fig. 2.31: An example of a package with a gain of +12 to +12dB: the half-inch particle velocity probe (1998).

Measuring sound intensity

Sound intensity is useful for measurement of sound power, identification and ranking of sources, visualisation of sound fields, measurement of transmission loss, identification of transmission paths, etc. In the pioneering days in the 1970s and 1980s sound intensity was a hot research topic, but since then it has become a well-known and established method, described in several standards. The conventional measurement technique employs two matched condenser microphones. However, a particle velocity transducer called the 'Microflown' and an intensity probe based on this transducer combined with a small pressure microphone have recently become available.

The sound intensity in a specified direction at a point is the average rate at which sound energy is transmitted through a unit area perpendicular to the specified direction at the point considered. This measurement allows obtaining information about noise sources independent on the environment or other noise sources. It is thus a vector quantity; defined as the time averaged product of the sound pressure (a scalar quantity, p) and the corresponding particle velocity (a vector quantity, u) at the same position.

In the general case measurement of sound intensity requires the use of at least two transducers. Sound intensity is usually measured with the p-p method, which combines two pressure micro-phones and makes use of a finite-difference approximation to the pressure gradient. The alternative p-u

method involves combining a pressure transducer with a particle velocity transducer.

The p-p method is based upon Newton's second law that is the equation of motion:

$$\rho \cdot \frac{\partial u_r}{\partial t} = -\frac{\partial p}{\partial r} \qquad (2.26)$$

Where u_r is the particle velocity in one direction r. Since the pressure gradient is proportional to the particle acceleration, the particle velocity can be obtained by integrating the pressure gradient in respect to time.

Fig. 2.32: A well-chosen package will result in a particle velocity gain (top view).

In practice, the pressure gradient is approximated by measuring the pressure at two points at a close distance Δr.

$$u_r = -\frac{1}{\rho \Delta r} \int (p_a - p_b) dt \qquad (2.27)$$

This approximation is valid as long as the separation, Δr, is small compared with the wavelength. If the frequency increases the spacing should be chosen smaller. In practice three spacings are being used: 5cm, 12mm and 6mm. A practical sound intensity probe consist of two closely spaced pressure microphones, as for instance the Brüel & Kjær intensity probe fitted with two ½" pressure microphones, see Fig. 2.32.

The p-p method is sensitive to large values of the pressure-intensity index, but not to high values of the reactivity. By contrast the p-u method is sensitive to high values of the reactivity, but not to large values of the pressure-intensity index. Thus p-u intensity measurement systems are potentially less affected by extraneous noise but more affected by reactive near fields than p-p intensity measurement systems. Both limitations can be serious in practical measurements, and one cannot conclude from these considerations that one method is superior to the other.

In measurements with p-p probes the acceptable pressure-intensity index depends on how well the two microphones are matched, that is, on the ‘pressure-residual intensity index’ of the probe [1]. With state-of-the-art equipment a pressure-intensity index exceeding 10dB is ‘high’ in most of the frequency range; below 200Hz even an index of 5dB is ‘high’. Such high values of the pressure-intensity index will occur if there is strong background noise from other sources than the one under test, in particular if the measurement takes place in reverberant surroundings. If the pressure-intensity index is unacceptably high possible countermeasures include moving the measurement surface towards the source under test, shielding the extraneous sources with temporary screens, and introducing more absorption in the room.

In measurements with p-u probes the acceptable reactivity depends on the accuracy of the phase calibration of the device (described in section 3). A reactivity of more than 5dB is ‘high’, but values of up to 25dB can occur [8]. Since the phenomenon is associated with near fields of sources the only remedy is to use a measurement surface further away from the source under investigation.

It is more complicated to calibrate the Microflown sound intensity probe than a p-p sound intensity probe. On the other hand the Microflown probe is much smaller than a p-p intensity probe, which is an advantage in many applications. With a p-u probe is easily possible to measure the 3D intensity and the bandwidth of the probes is better than the p-p probes.

Measuring vibration

In the previous paragraphs methods to measure sound where described. In this paragraph deals with the laser vibrometer and the accelerometer, the two most known methods to determine the vibration of a structure.

Fig. 2.33: An example of a laser vibrometer (range: 1μm/s to 5mm/s; bandwidth: DC to 25kHz, laser head dimensions: 50x25x20cm; Weight: 12kg).

A laser vibrometer determines the structural surface velocity with the use of a laser beam. It is a sensitive, non contact method that operates in a wide frequency bandwidth and large dynamic range. The measurement distance may be large (up to meters). It is possible to scan a large number

of points with a scanning laser vibrometer. Disadvantages of a laser vibrometer are its large size, a stability requirement, it high price and that it is sometimes difficult to use.

An accelerometer is a low cost sensor that is sensitive for the acceleration of a structure (so the time derivative of the velocity). The sensor is broad banded and has a large dynamic range. Disadvantage is that the sensor must be fixed to the structure which is time consuming and may alter the structure to be measured.

Fig. 2.34: An example of an accelerometer (8mm x 11mm, 2gram in weight, bandwidth 1.5Hz to 18kHz). Right a three dimensional accelerometer.

It is possible to measure the vibration with Microflowns. See further chapter 7: ‘vibration measurements’

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