ACOUSTIC IMPEDANCE MEASUREMENT, PART I: A REVIEW

J.-P. DALMONT

Institut d’Acoustique et de mécanique, Laboratoire d’Acoustique de l’Université du Maine (UMR CNRS 6613), Avenue Olivier Messiaen, 72085 Le Mans Cedex 9, France.
E-mail: jean-pierre.dalmont@univ-lemans.fr

(Received 9 March 2000, and in final form 4 October 2000)

In this review, impedance sensors are presented as linear sensors with two entries whose signals are influenced by both pressure and volume velocity. Pressure is generally measured with a microphone, therefore impedance sensors are classified according to the way the volume velocity is determined or controlled. The review touches on the multi-port characterization which is shown to be a generalization of impedance measurement. Finally the question of the calibration and error sources is discussed.

© 2001 Academic Press

1. INTRODUCTION

Characterization of acoustic passive elements is necessary for many applications such as wind instruments, horns, mufflers, absorbing materials, vocal tract, ear canal or lumped elements. Initially, impedance sensors were analogue devices, impedance curves being obtained through a direct plotting of an analogue output signal [1–5]. Most of these impedance sensors then used a source of constant velocity and a microphone giving a pressure signal approximately proportional to the impedance measured. Since 1975 [6, 7], and because the development of calculators allows a computation of the data, other systems based only on pressure measurements have become widespread [8–11]. These systems have been used for impedance of absorbing materials [12] as well as for multi-port characterization [13–15].

The aim of the present paper is to give a synthetic overview of the literature on impedance measurements and to discuss the advantage and disadvantage of each measurement method. Prior to the review a general theory of impedance measurement is given in section 2. An impedance sensor is defined as a system involving two transducers whose input signals are linearly related to pressure and volume velocity. A concept of “response” matrix which relates these input signals to pressure and volume velocity at the input of the measured system is introduced. It is shown that the calibration of a sensor leads to three complex calibration functions which can be either theoretically determined or deduced from a calibration. The characterization of multi-ports can be seen as a generalization of impedance measurement. Consequently, the review in section 3 presents together the setups for multi-port characterization and the setups for input impedance measurements. Despite calibration having been the subject of only a few papers, although it is discussed partly in some papers [16–21], it is an important point which governs the accuracy of the measurement setups. Finally, two main strategies for the calibration are presented in section 4: a partial calibration in which the transducers of the set-up are individually
calibrated and a “complete” calibration in which the three calibration functions are
determined. The main error sources are discussed in section 4.4.

2. GENERAL CONSIDERATIONS

Impedance can be considered as the response of a particular passive system to a harmonic
excitation. In acoustics the specific impedance is defined as the ratio of the pressure and the
particle velocity at a given point. As the particle velocity is a vectorial quantity the
impedance depends on the direction of this vector. This quantity is mainly used in free field
or in cavities. In tubes, when considering only the plane wave mode the following definition
is preferred: the acoustical impedance \( Z \) at any cross-section of the tube is the ratio of the
mean pressure \( P \) and the volume velocity \( U \) on the cross-section:

\[
Z = \frac{P}{U}.
\]

One may distinguish between input impedances, for which both quantities are considered
at the same point, and transfer impedances for which the two quantities are considered at
two different points. For a resonator, which can be seen as a one-port passive linear
acoustical system, the input impedance is sufficient to describe its behaviour. For a passive
linear \( N \)-port element there is a relation between the pressure \( P_i \) on each side and the
volume velocities \( U_j \), where \( i \) and \( j \) are port numbers \((i, j = 1 \text{ to } N)\), which can be written in
a matrix form:

\[
\{P_i\} = [Z_{ij}] \{U_j\},
\]

where \( \{P_i\} \) and \( \{U_j\} \) are \( N \)-vectors containing the pressure and volume velocities on each
port. \([Z_{ij}]\) is the \( N \times N \) impedance matrix of the system. \( Z_{jj} \) terms are the ‘self’ impedance
terms, i.e. the input impedances of the port \( j \) when all other ports are closed with an infinite
impedance \((U_i = 0 \text{ for } i \neq j)\). \( Z_{ij} = P_i/U_j \) are the ‘mutual’ impedances i.e. the transfer
impedances when all ports are closed except port \( j \) \((U_i = 0 \text{ for } i \neq j)\). The characterization of
a \( N \)-port then implies, in the more general case, measurement of \( N \) input impedances and
\( N(N - 1) \) transfer impedances. The measurement of the self impedance on each port implies
the determination of the ratio of pressure and volume velocity. For determining these two
quantities two transducers are needed on each port. Each pair of transducers can be
considered as an impedance measurement set-up. For a \( N \)-port characterisation
\( N \) impedance measurement set-ups and \( N \) independent states are needed [14, 15, 22–28].
One can choose to use each source either separately or simultaneously. The choice of the
states depends partly on the measured quantity. For example, for a two-port, a solution is to
do one measurement with both sources in phase and another one with both sources
opposite in phase [27]. Another idea is to use one source to realise an active anechoic
termination [22, 23]. With this method the reflection and transmission parameters of the
\( N \)-port, that is the scattering matrix, are obtained directly. Since all measurements cannot
be done simultaneously, the same set-up can possibly be used for different ports. Another
solution using only one source is to change the location of the source [28] or the load on the
ports where there is no source [24, 25, 29].

The choice of the excitation signal depends on the signal-to-noise ratio, the frequency
resolution and the time available for the measurement. The best signal-to-noise ratio is
obtained with a slowly swept sinus and a lock-in amplifier, but measurements take more
time than with white noise or chirps. Pulses can also be used, especially for time domain investigations. MLS signals can also be used.

To reduce errors, an over-determination can be obtained by doing more measurements than strictly needed. This is especially advised in multi-microphones techniques. As a pair of microphones is generally not sufficient to give good results over a large frequency range, more than two microphones are needed. The different measurements can then be computed using a least square method on well chosen quantities [14, 15, 29–31].

Consider now the case of a given port instrumented with two transducers. Let $e$ and $u$ be the input signals of the transducers. For a passive linear set-up $e$ and $u$ are related to pressure $P$ and volume velocity $U$ at the reference section by the relation:

$$
\begin{pmatrix}
e \\
u
\end{pmatrix} = \begin{pmatrix}
m & n \\
p & q
\end{pmatrix} \begin{pmatrix}
P \\
U
\end{pmatrix},
$$

where $\begin{pmatrix}
m & n \\
p & q
\end{pmatrix}$ is the “response” matrix which characterizes the two transducers and the way they are coupled to the port. Equation (3) suggests the sensor whose signal is $e$ is preferentially devoted to pressure and the sensor whose signal is $u$ is devoted to volume velocity. This is not necessary. One of the transducers can be a source, the corresponding signal being the excitation signal. Introducing the impedance $Z = P/U$ and dividing the two equations of (3) gives the ratio of the two signals $e/u$ [18, 32, 33]:

$$
e = R \frac{(Z + \beta)}{(1 + \delta Z)},
$$

where $R = m/q$, $\beta = n/m$, $\delta = p/q$ are complex and frequency dependent parameters, and characteristics of the sensor. $R$ can be called the first order response of the sensor. Calibrating the sensor amounts to determining these three parameters. However, determination of $R$, $\beta$ and $\delta$ is not sufficient for transfer impedance measurements because in that case two transducers of two different impedance set-ups are used. A solution is then to perform an absolute calibration of one of the sensors or to use a specific method for transfer impedance measurement calibration, as proposed in Part II of this paper.

### 3. OVERVIEW OF VARIOUS TECHNIQUES

Any transducer measuring any physical quantity which is linearly related to pressure or volume velocity can be used for impedance measurement; thus many systems can be built. They differ initially in the choice of the measured quantities and later in the way these quantities are measured. An overview of various techniques has already been given by Benade & Ibisi [34]. The aim of this section is to extend this overview. The different techniques can be classified according to the measured quantities.

#### 3.1. SET-UPS USING ONE PRESSURE AND ONE VOLUME VELOCITY TRANSDUCER

The most straightforward way to obtain an impedance on a reference plane is to measure the pressure and the volume velocity on this reference plane simultaneously (see Figure 1). In this case the signal $e$ given by the microphone and the signal $u$ given by the volume velocity transducer are, respectively, proportional to the pressure $P$ and the volume velocity $U$. 
The characteristic matrix of the impedance measurement set-up is then simply given by:

\[
\begin{pmatrix}
m & n \\
p & q
\end{pmatrix} = \begin{pmatrix} R_e & 0 \\ 0 & R_u \end{pmatrix},
\]

where \( R_e \) and \( R_u \) are the respective responses of the two transducers. In this ideal case:

\[
\frac{e}{u} = \frac{R_e}{R_u} Z,
\]

Thus the first order \( R \) response is equal to \( R_e/R_u \) and the two parameters \( \beta \) and \( \delta \) are equal to zero. Impedance is proportional to the signal ratio. In practice this cannot be achieved exactly, but in methods based on this principle the two parameters \( \beta \) and \( \delta \) remain small and can be considered as corrective or error terms. However, at ‘resonance’ frequencies (maximum of the modulus of the impedance) the effect of \( \delta \) can be rather large which leads to an error in the determination of the frequency of the resonance and its amplitude. \( \delta \) is mainly related to the mechanical impedance of the source and of the microphone which can both be modelled as an added equivalent volume (see for example [30, 32, 33, 35]). \( \beta \) is mainly related to higher order mode effects or to the distance between the source and the microphone. Its effect is more important for low impedance. The measurement of the pressure is conveniently carried out using a microphone, but the measurement of the volume velocity is more difficult. The velocity can be measured, for example, by a hot wire anemometer as was successfully done by Pratt et al. [36, 37] but this remains unusual. Recently de Bree et al. [38] constructed an acoustical velocity sensor which could be used to build such an impedance sensor. However the easiest way is to use a source of known volume velocity. Various sources have been used: a shaker with a piston [39–43], an ionophone [44], a diaphragm excited with a loudspeaker [45], a piezoelectric transducer [34] or a capacitive microphone cartridge used as a source [21, 46]. In these measurements the stability and the magnitude of the volume velocity can be evaluated by carrying out a calibration or another measurement such as a displacement [45], an acceleration [40–43] or another pressure. This last case is discussed in the following section 3.2.
3.2. SET-UPS USING TWO PRESSURE TRANSDUCERS, ONE PRESSURE BEING PROPORTIONAL TO THE VOLUME VELOCITY

In this section set-ups are described which can either be considered as systems with known volume-velocity source or as two-microphone systems. These are designed so that one of the microphones measures a pressure proportional to the volume velocity. The most obvious acoustic source is the loudspeaker. As the impedance of a loudspeaker is usually relatively low compared to the impedance of resonant loads it cannot be considered as a truly constant volume velocity source. To measure the volume velocity produced by the loudspeaker the back of the loudspeaker can be enclosed and the pressure inside the enclosure can be measured (see Figure 2). For low frequencies the pressure in the enclosure is uniform and proportional to the volume velocity [43, 47]. The range of such a setup is then limited to frequencies below the first resonance of the cavity. Another possibility is feeding a capillary tube, which has a high acoustic impedance, to a loudspeaker (see Figure 3). The other end of the tube acts like a volume velocity source. The pressure in the matching volume between the loudspeaker and the capillary tube is, if the resistance is high enough compared to the measured impedance, proportional to the volume velocity in the tube. A large number of such impedance sensors have been build since Webster [1] (see for example [3–5, 48, 49]) and some papers discuss the optimization of these sensors [32, 50, 51]. Parameter $\delta$ of equation (4) is equal to the input admittance of the capillary tube. To make this term negligible the pressure gap between the input and the output of the capillary tube has to be large even for large impedance. A consequence is that the vibrations induced
by the loudspeaker can have a large influence on the microphone signal, especially for small impedances. To reduce this influence it is good to build a massive set-up mechanically dissociated from the source [23].

Techniques described in sections 3.1 and 3.2 can be considered as direct measurements because the impedance is in first approximation proportional to the ratio of the two measured quantities. Moreover a feedback loop was often used to keep the second quantity constant. Then the direct plotting of \( e \) gives a curve approximately proportional to the impedance. This is the main difference with the two-microphone technique, described in the following section, in which impedance is obtained through computation of the data. This is why the two-microphone technique has been only developed since the 1980s.

3.3. SET-UPS USING TWO PRESSURE TRANSDUCERS

As the velocity is locally proportional to the pressure gradient it is possible to use a pair of microphones to deduce the volume velocity (see Figure 4). Methods using a pair of microphones have often been used since the 1970s for impedance measurements of absorbing materials and for matrix measurements [6–15, 29]. Any design of the sensor can be used [18] but in most cases microphones are located along a straight tube. The calibration is then simplified because the propagation between two microphones can be theoretically calculated.

This method is very different from the set-up with a known volume velocity source mainly because the relationship between the two pressure signals, the pressure \( P \) and the volume velocity \( U \) is not as simple as in the first technique. However, a pair of microphones on a straight tube can be considered as a sensor measuring the impedance at the middle of the two microphones. Considering the quantities \( (p_1 + p_2)/2 \) and \( (p_1 - p_2)/2 \) as auxiliary measured quantities, it can easily be shown that these are related to the pressure \( P_0 \) and volume velocity \( U_0 \) at the midpoint between the two microphones (see Figure 4) by the following relation:

\[
\begin{pmatrix}
\frac{p_1 + p_2}{2} \\
\frac{p_1 - p_2}{2}
\end{pmatrix} =
\begin{pmatrix}
\cosh \Gamma L & 0 \\
0 & Z_C \sinh \Gamma L
\end{pmatrix}
\begin{pmatrix}
P_0 \\U_0
\end{pmatrix},
\]

where \( Z_C \) and \( \Gamma \) are, respectively, the characteristic impedance and the propagation constant and \( L \) half of the distance between the microphones. It appears that this is
equivalent to a set-up with a pressure sensor whose response is \( \cosh \Gamma L \) and a volume velocity sensor whose response is \( Z_c \sinh \Gamma L \), the reference abscissa being at the mid-distance between the microphones. It appears that two-microphone sensors are theoretically not so different from known volume velocity transducers.

The two-microphone method can give satisfactory results but not for all frequencies: measurements are not possible if the distance \( 2L \) between the two microphones is too small \( (\sinh \Gamma L \ll 1) \) or close to a multiple of the half wavelength. This can be overcome by using more than two microphones. Some papers analyze the error sources in this technique [16, 17, 52].

3.4. SET-UPS USING A SINGLE PRESSURE TRANSDUCER

Because the calibration of two microphones with respect one to another can involve errors, some authors prefer to use only one microphone in two different measurements. These two measurements can be obtained by changing the location of the microphone [53, 54] as in the classical Kundt tube technique [55] or by changing the load [56, 57]. An elegant solution is to do only one measurement using a pulse. If the source is far enough from the microphone the incoming and outcoming wave can be separated in the time domain giving two separate signals. After a Fourier transform the impedance is obtained. This method, called pulse reflectometry, is mainly used for bore reconstruction of musical wind instruments [58–60] and tracheal areas [61, 62].

3.5. SET-UPS USING VARIOUS TRANSDUCERS

Transducers other than pressure and velocity transducers can be used. The restriction is that the quantities are linearly related to pressure and volume velocity. As an example, some authors build a setup in which the impedance of the load is deduced from the electrical impedance of the loudspeaker [63, 64]. Another unusual example is given in reference [65]: quite accurate measurements are obtained using a technique similar to the two-microphones technique in which the microphones are replaced by two velocity sensors.

4. CALIBRATION AND ERRORS

4.1. COMPLETE OR PARTIAL CALIBRATION

The “complete” calibration has to be distinguished from a partial calibration. In a “complete” calibration the sensor is considered as a black box and the only assumption is the stability and the linearity of the sensor. In a partial calibration a model of the sensor is postulated for which the parameters must be determined. For a “complete” calibration, as the impedance is related to the ratio between any two signals by three complex functions of frequency (equation (4)), three known loads have to be measured from which these calibration functions can be deduced. This is the basic principle of the two-microphone-three-calibration (TMTC) method [18]. In fact calibration is not total because the influence of quantities such as temperature is not taken into account, the calibration being theoretically carried out for a given temperature. The stability of the sensor is difficult to maintain between different measurements especially due to temperature fluctuations which

IMPEDEANCE MEASUREMENT REVIEW 433
may change the value of the calibration parameters. Temperature must be controlled with
great accuracy during the calibration and following measurements.

Because a total calibration is sometime difficult and not always needed, a partial
 calibration is often done. This is possible if a theoretical model of the sensor is available. Then only the parameters of the model have to be determined. In the two-microphone
 technique a common procedure is to calibrate the relative response of the two microphones
 and to assume that the propagation between the two microphones is known. With
 this method it is easier to achieve good calibration. However, it remains necessary
to control the temperature with great accuracy. To avoid temperature measurements
the wave constant in the tube between the microphones can be determined using additional
microphones. In the known volume velocity source technique the first order response is
obtained from a calibration. The other parameters, including higher modes effects and
the impedance of the source, are most often neglected (on this subject see refer-
ences [30, 32, 35]). If the first order response is assumed to be constant no calibration
is needed. In practice this is sufficient when only the frequencies of the main resonances are
of interest.

4.2. THE CHOICE OF THE KNOWN LOADS

Various loads have been used. To build a load of known impedance the main criterion is
that the analytical expression of its impedance involves a reduced number of physical
parameters so that its value can be accurately determined theoretically. Two main shapes
have been used: cavities and tubes. Cavities can be used for low frequencies only, because
for higher frequencies higher modes must be taken into account: the amplitudes of
these modes are difficult to determine theoretically [2, 48]. Straight tubes are most often
used. Open tubes can be used but the radiation impedance of the open end is not known
with a high accuracy and depends on the wall thickness of the tube [66]. Moreover some
noise can penetrate through the open end of the tube. So a closed tube is a better choice
because the end impedance is known [56] and can often be assumed to be infinite. A long
tube is attractive because it gives many resonances and antiresonances and permits
a calibration over a large dynamic range for closely spaced frequencies. For a total
calibration, three tubes of different lengths have to be employed [18]. This is rarely done
and usually, considering that for a long tube, high impedances alternate with low
impedances, only one tube is measured. This is sufficient when calibration parameters vary
slightly with frequency so that they can be obtained by interpolation between two
resonances of the tube.

4.3. CALIBRATION PROCEDURE BASED ON THE RESONANCE ANALYSIS OF A SINGLE TUBE

Different methods based on the measurement of a tube have been derived. Bruneau [21]
derived a “complete” calibration method based on the properties of the hyperbolic tangent
function. The most often used method is a method based on the resonance analysis. This
method has been most often used only partially. The derivation of a complete calibration
with this method is now shown.

The input impedance of a closed tube of length $L$ is given by $Z = Z_c \coth[\Gamma L + \arctanh(Z_c Y_t)]$ with $Z_c$ the characteristic impedance and $\Gamma = jk + (1 + i)\alpha$ the
propagation constant, $\alpha$ being the attenuation constant. Assuming $\arg\tanh(Z_c Y_t) \approx 0$ and
$Z_c \approx \rho c/\mathcal{S}$ and setting $c' = c/(1 + \alpha/k)$, resonance frequencies $f$, amplitude $Z_{\text{max}}$ and
Q-factor $Q$ are given by (see reference [33]):

$$f = \frac{nc'}{2L}, \quad Z_{\text{max}} = \frac{Z_c}{\pi L} \quad \text{and} \quad Q = \frac{k}{2\pi}. \quad (8)$$

where $n$ is a strictly positive integer.

Similarly the antiresonances are given by

$$f = \frac{(n - 1/2)c'}{2L}, \quad Z_{\text{min}} = Z_c\pi L \quad \text{and} \quad Q = \frac{k}{2\pi}. \quad (9)$$

where $n$ is a strictly positive integer.

For the maxima of the measured quantity $e/u$, assuming $\alpha L \ll 1$, $\delta Z_C \ll 1$ and $\beta/Z_C \ll 1$, the corresponding quantities are given by:

$$f = \frac{nc'}{2[L + \text{Im}(\delta Z_C)/\pi]}, \quad \left(\frac{e}{u}\right)_{\text{max}} = RZ_c \frac{1}{\alpha L + \text{Re}(\delta Z_C)} \quad \text{and} \quad Q = \frac{kL}{2[\alpha L + \text{Re}(\delta Z_C)]}. \quad (10)$$

Similarly the antiresonances are given by:

$$f = \frac{(n - 1/2)c'}{2[L + \text{Im}(-\beta/Z_C)]}, \quad \left(\frac{e}{u}\right)_{\text{min}} = RZ_c(\alpha L + \text{Re}(\beta/Z_C)) \quad \text{and} \quad Q = \frac{kL}{2[\alpha L + \text{Re}(\beta/Z_C)]}. \quad (11)$$

When comparing the theoretical impedances with the measured values of $e/u$, the shifts of the frequencies of the maxima give the imaginary part of parameter $\delta$ while the shifts of the frequencies of the minima give the imaginary part of parameter $\beta$. The differences in the quality factors of, respectively, the resonances and antiresonances give the real parts of, respectively, $\delta$ and $\beta$. The differences in the amplitudes of resonances and antiresonances, after corrections due to the real parts of $\beta$ and $\delta$, give the modulus $|R|$ of the first order response. Finally the phase shifts at the maxima and minima give the phase response (see reference [33]). The frequency dependence of the modulus corresponds to what Benade and Ibisi [34] call the mean line of the dB plot amplitude. The determination of this mean line is not so easy because accurate measurements of the minima are sometimes difficult when the signal-to-noise ratio is low. To avoid this difficulty Benade and Ibisi [34] propose to reduce the dynamics of the resonances by putting some absorbing material in the tube. However, it was found here that the accuracy of the calibration is not very good because the mean line does not always correspond exactly to the characteristic impedance of the tube [33]. The main problem of the calibration procedure based on resonance frequency analysis is that it implies the use of a long tube in which the temperature must be accurately known. In addition, accurate measurements of the $Q$ factors are difficult. This method, though it can give accurate values of the calibration parameters, has mainly been used only to obtain the first order response and eventually the length correction at the input for resonance frequencies (related to the imaginary part of $\delta$).

4.4. ERRORS

Some errors can partly be corrected by a calibration; that is, they can be included in the calibration parameters. On the other hand calibration implies the use of known loads,
knowledge of which is corrupted by errors which can be either geometrical errors or temperature errors. If only the sensors are calibrated their calibration also induces errors. Thus, the final error is the sum of measurement errors and calibration errors.

Temperature has a direct influence on the speed of sound and therefore on the impedance of the calibration loads. As a consequence, during the calibration the temperature has to be accurately measured so that the assumed value of the speed of sound is not too different from the actual one. This is especially true when the known load is a long tube. For example using a tube of length $L = 1\text{ m}$, a $1^\circ\text{C}$ error in temperature is equivalent to an error of 1.7 mm on the length of the tube. To limit this error a short tube can be measured. This allows the determination of the wave constant as presented in Part II of the paper. However, temperature must be stable during the measurement of the calibration loads. Temperature can also have an influence on the stability of the calibration parameters. This is especially critical in the two (or more) microphones technique in which the number of wavelengths between the microphones is directly related to the velocity of sound. This is the main problem with this technique [16, 17, 52] and probably the reason why a total calibration is rarely done.

Calibration supposes that the geometry of the “known” loads or the distances between the sensors are known exactly. If the measurements of the length of the calibration loads and of the distances between the sensors are not precise enough, the accuracy may be limited. The operation consisting of disassembling–reassembling pieces of a set-up should be limited because they could introduce some fluctuations in these parameters.

As impedance sensors are intended to measure the plane wave mode only, higher order modes effects can be considered as errors or corrective terms. For sensors with sources of known volume velocity the influence of the higher order modes can be included in the term $\beta$ [32, 35, 46]. The effects of higher modes are negligible if the impedance is sufficiently high which is usually the case for resonance frequencies ($Z \gg \beta$). If only resonance frequencies are needed this can be often neglected. However, for a “complete” calibration these effects must be taken into account. The difficulty is that the effect of higher order modes depends on the coupling between the resonator and the sensor. Thus, even if these effects have been precisely determined during the calibration, they might be different for the measured load. Therefore, the only solution is to conduct a calibration with the same coupling as for the measurement. If the resonator has a given input diameter and is fixed in a certain manner on the sensor, the calibration tubes must have the same diameter and be fixed in the same way. This implies that calibration cannot be definitive and valid for every load. This makes a complete calibration complex. If a systematic calibration is not intended, one solution is to try to minimize the higher order mode effects. Since the first higher order mode is the first helical mode, it is preferable to build a set-up which is symmetrical in the measurement plane [46, 33].

Linearity and stability are the basic hypotheses in the present discussion. It is possible to detect non-linearity by measuring at different levels but this may not be sufficient. The stability can be detected by repeating calibrations at different times and comparing calibration results. Also, calibration can be checked by measuring a supplementary known load not used for calibration. Lavrentjev et al. [67] give a method for detecting non-linearities when using the least square method.

ACKNOWLEDGMENTS

The author thanks Kees Nederveen and Murray Campbell for the reviewing of this paper and Véronique Dubos for her work on the bibliography.
REFERENCES


