Indirect measurement of the bulk properties of acoustic absorbing samples

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Abstract

In this paper, a new procedure is investigated to measure bulk properties of acoustic absorbing samples that are not clamped on the sides. Tests have been performed with a PU probe and with a sound source that is emitting spherical sound waves above a large slab of a homogeneous material. Multiple tests have been performed for different configurations, i.e. with and without a backplate as well as with two sample thicknesses, for several sound source heights. A method is presented that uses a combination of tests performed under different conditions in order to obtain the complex characteristic impedance and complex wavenumber of the sample. The method utilizes a plane wave model to calculate the bulk properties for each sound source height. Errorneous estimates are found since this model does not correct for spherical waves. However, depending on the height of the sound source the results are affected by near field effects differently. Tests at different heights are combined, and the plane wave values are obtained using an extrapolation technique. The results of the method are investigated through tests and simulations.

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1 Introduction

There are several methods to measure the surface impedance, the reflection coefficient, and the absorption coefficient of acoustic absorbing samples in a particular configuration. If the bulk properties of the sample (i.e. the complex characteristic impedance \( Z_1 \) and complex wavenumber \( k_1 \)) are measured, they can also be determined for different sample configurations [1, 3]. There are two well known methods to measure bulk properties. Firstly, \( Z_1 \) and \( k_1 \) can be obtained using a Kundt tube by taking impedance measurements on both sides of the sample. In addition, they can also be determined indirectly by measuring separate sample properties (such as the flow resistivity, porosity, tortuosity, etc.), and using one of many available material models, see [4 5 10].

Perhaps the most popular model is the empirical formula of Delany and Bazley from 1969 for fibrous materials [12, 13, 14] pp. 20-23. In this model an expression for \( Z_1 \) and \( k_1 \) values as a function of frequency is derived which only requires the flow resistivity \( \sigma \). Figure 1 shows the real and imaginary parts of \( Z_1 \) that are normalised with the characteristic impedance in air \( (\rho c \approx 412 \, \text{Pa} \cdot \text{s/m}) \), and the real and imaginary parts of \( k_1 \) that are normalised with the characteristic wavenumber in air, for an arbitrary flow resistivity of \( \sigma = 12 \, \text{kPa/m}^2 \). Appendix A contains the model expressions, a description of the boundaries within which this model is valid, and the simulated characteristic impedance and wavenumber values for several values of \( \sigma \).

![Figure 1: Normalised characteristic impedance and wavenumber calculated using the Delany and Bazley model for \( \sigma = 12 \, \text{kPa/m}^2 \)](image)

In this paper, another method to measure the bulk properties is considered which could potentially be used in situ, or even with a hand-held set-up. The method used is an expansion of a well-studied in situ technique to determine the absorption coefficient of samples from an impedance above the sample, see [7 11 15 16 2]. In this method, a sound source that emits spherical waves is positioned above a probe which measures the impedance near the sample sur-
face. For convenience, the probe and the sound source are usually combined in a hand-held set-up. With a spherical sound field model the reflection coefficient and, given that there is no transmission through the sample, the absorption coefficient, can be obtained. Here, such impedance measurements are used to determine the bulk properties $Z_1$ and $k_1$ of the sample. For this purpose, the sound field above and inside the sample is modelled. One complex value can be obtained with only a single measurement, which makes it impossible to determine both $Z_1$ and $k_1$. However, it is possible to determine both quantities when results from multiple measurements are combined. A method will be introduced that makes use of different sample configurations; samples with two thicknesses, and with or without a backplate.

The set-up and test conditions are described in the following chapter. Subsequently, the method comprising a plane wave model to calculate the bulk properties of the sample will be introduced. Measurements at different source heights will be affected by near field effects differently since the model does not correct for the propagation of spherical sound waves. A similar method to the one introduced in [16] is used to extrapolate the results from several sound source heights to $r = \infty$. Thereafter, the test results are shown. Finally, a spherical wave model is used to investigate the method through simulation.

### 2 Measurement set-up

![Figure 2: Probe and sound source above of the sample (without a backplate)](image)

The measurement set-up used for the experiments is shown in figure 2. The acoustic absorbing sample measured is made from a single homogeneous slab of open melamine foam called Flamex. The measurements were performed using a PU probe containing a sound pressure sensor and an acoustic particle velocity sensor. With such a probe, quantities such as intensity, impedance and energy can be obtained directly.

A point source is required for the spherical sound field model used in this paper. However, we make use of a so-called piston-on-a-sphere sound source instead, mainly due to its high broad band radiation efficiency compared to that of typical monopole loudspeakers. The sound source consists of a loudspeaker, which is mounted inside a small rigid sphere. In [8] and [14] it is shown that there are no significant edge effects with such a loudspeaker, and that the radiation is almost omni-directional at low frequencies. It was also found that the free field radiation impedance in the axis of the loudspeaker membrane is similar to that of a point source. For example, for our piston-on-a-sphere loudspeaker, with a sphere radius of 50.5 mm and a piston radius of 37.5 mm, the impedance deviation from that of a monopole is only $\pm 0.36$ dB in amplitude and $\pm 1$ degrees in phase in a $10\text{Hz} - 10\text{kHz}$ band, if the spacing between the probe and the loudspeaker is larger than 0.1 m.

### 3 Description of the experiments

![Figure 3: Three measurement arrangements](image)
Measurements have been performed for:

- different sample configurations, i.e. without a sample, from above a sample with a backplate (the sample is placed on the acoustically hard floor of the room), and above a sample without a backplate (the sample is suspended so that there is air on both sides), see figure 8,
- two sample thicknesses: $d_1 = 0.027\,m$ and $d_2 = 0.054\,m$;
- seven sound source heights that are realistic for a hand-held set-up: $r = 0.15$, 0.20, 0.25, 0.30, 0.35, 0.40, and 0.60\,m.

The results of the individual measurements are combined in order to obtain the bulk properties. It is assumed that the bulk properties of the sample remain the same for different mounting conditions.

The tests were performed with a miniature PU probe, which incorporates a sound pressure sensor and a particle velocity sensor. The probe operates in a wide frequency range (20 Hz and 20 kHz). The influence on the sound field is low due to its small width of approximately 3\,mm. The distance between the centre of the probe and the sound absorbing sample was small ($h = 0.005\,m$). The measured impedance can therefore be assumed to be similar to the surface impedance of the sample. Maybe it would be possible to use larger probes. However, the influence of using larger distance between the centre of the probe and the sample surface should then be investigated, which might be done in a future study. The size of the sample is $1.5\,m^2$, which can be considered sufficiently large; in other studies it was shown that the minimal sample size ranges from 0.08 $m^2$ to 0.36 $m^2$ for frequencies higher than 100 Hz, depending on the geometries of the set-up and on the assumptions that were made of the sound field [6, 4, 13].

The tests were performed in a quiet room with some sound-dampening materials on the walls. A moving average procedure described in [15] was used to cancel the remaining parasitic room reflections. The procedure is based on the principle that reflections have a harmonic modulation effect on sound pressure or particle velocity in the frequency domain. Their influence depends on the distance to the reflective surface and the strength of the reflected wave. When there are many surrounding surfaces at different distances and directions there are many interfering reflecting waves. Although a small frequency band can be strongly affected by reflections from a single direction, the average of multiple adjacent frequency bands are influenced less by individual acoustic modes. A moving average of the frequency response can therefore be applied to minimise the effect of reflections. Effectively, the frequency response is smoothed. Therefore, the application of this technique is only allowed when the impedance of the material is expected to vary gradually with frequency. Furthermore, the level of the direct sound from source should be strong compared to the sound level of reflections from surrounding objects. The latter condition is met during our experiments because the distances between the sound source, the probe and the sample are kept small.

### 4 Determine bulk properties

In this chapter, a method is presented to calculate the bulk properties of the sample while using a plane wave model. The results from different sound source heights are extrapolated to $r = \infty$ (as if there are plane waves) because a spherical sound field is present during the test. This section contains a description of the model used, the sensor calibration procedure, the method to determine the impedance above the sample, and the extrapolation procedure.

#### 4.1 Plane wave model

There are expressions to relate the surface impedance to the bulk properties of the sample if there are plane waves and the sample is homogeneous. At normal incidence, the impedance above such a sample with and without a backplate can be calculated using the complex characteristic impedance $Z_1$ and complex characteristic wavenumber $k_1$ [16], [4] pp. 18-19, [9] p. 153 and 284:

$$Z_{\text{backplate}} = -iZ_1\cot(k_1d) \quad (1)$$

$$Z_{\text{nobackplate}} = Z_1\frac{iZ_1\tan(k_1d) + \rho c}{\tan(k_1d)\rho_c + Z_1} \quad (2)$$

#### 4.2 Calibration

The ratio of sound pressure to particle velocity measured above the sample cannot be compared directly to the theoretical impedance values because the amplitude and phase responsivities of the sensors and the electric properties of the processing unit are unknown. Several calibration procedures exist to determine the responsivities, but these are quite elaborate. Instead, a measurement $Z_{\exp 0}$ without sample (see figure 8, configuration 0) is used as reference because the free field impedance $Z_{ff}$ is known from
theory [8], [9] p.128:

\[ Z_{ff} = \frac{ik_0r}{ik_0r + \rho c} \]  

(3)

The responsivities of the sensors are eliminated from the ratio of the measured impedance near the sample to the impedance in the free field. The actual impedance near the sample is then obtained by accounting for the free field impedance present during the measurement without sample. There are four impedances that can be determined because the sample was measured in four situations:

- with a backplate and thickness \( d_1 \); \( Z_{m1,d1} = \frac{Z_{exp,d1}}{Z_{exp,0}} Z_{ff} \)
- with a backplate and thickness \( d_2 \); \( Z_{m1,d2} = \frac{Z_{exp,d2}}{Z_{exp,0}} Z_{ff} \)
- without a backplate and thickness \( d_1 \); \( Z_{m2,d1} = \frac{Z_{exp,d1}}{Z_{exp,0}} Z_{ff} \)
- without a backplate and thickness \( d_2 \); \( Z_{m2,d2} = \frac{Z_{exp,d2}}{Z_{exp,0}} Z_{ff} \)

The advantage of performing a reference measurement with the same set-up is that no corrections are required for conditions like data acquisition settings, software settings, and the value of the impedance in air (which depends on temperature for example) because all of them are expected to remain constant during the period of the tests.

### 4.3 Determination of the bulk properties

The impedance near the sample can then be compared to the equations of the corresponding sample configuration. If different sample configurations are combined \( Z_1 \) and \( k_1 \) can be obtained using equation [1] and [2]. There are six possible combinations of two measurements:

- \( Z_{m1,d1} \) & \( Z_{m1,d2} \): sample with a backplate, two thicknesses;
- \( Z_{m1,d1} \) & \( Z_{m2,d1} \): sample with and without a backplate, thickness \( d_1 \);
- \( Z_{m1,d1} \) & \( Z_{m2,d2} \): sample with a backplate thickness \( d_1 \), and sample without a backplate thickness \( d_2 \);
- \( Z_{m1,d2} \) & \( Z_{m2,d1} \): sample with a backplate thickness \( d_2 \), and sample without a backplate thickness \( d_1 \);
- \( Z_{m1,d2} \) & \( Z_{m2,d2} \): sample with and without a backplate, thickness \( d_2 \);
- \( Z_{m2,d1} \) & \( Z_{m2,d2} \): sample without a backplate, two thicknesses.

The solutions for each pair of sample configurations are shown in appendix B.

### 4.4 Plane wave extrapolation

The outcome of plane wave models themselves is incorrect because no corrections for the propagation of spherical sound waves are made. In [16] a method was introduced to obtain the plane wave absorption coefficient from tests that were performed with several sound source heights. It was shown that the ratio of active absorbed to active ingoing intensities measured for a specific source height was equal to the absorption coefficient plus a near field term as function of \( r \). Each of these measurements is affected by the near field effects differently. By extrapolating the results from three or more measurements to \( r = \infty \) the plane wave absorption coefficient could be obtained.

Here, a similar extrapolation procedure is used to correct the results of the impedance measurements. For each frequency a polynomial fit versus \( r \) of the order 2 is made of the measured impedance, of \( Z_1 \), or of \( tan(k_1 d) \). For instance when the measured impedance is extrapolated towards the value \( r = \infty \) at least three measured values of \( Z_1(\frac{1}{r}) \) are used to solve the polynomial coefficients \( P_0 \), \( P_1 \) and \( P_2 \) in the expression:

\[
P \left( \frac{1}{r} \right) = P_2 \left( \frac{1}{r} \right)^2 + P_1 \left( \frac{1}{r} \right) + P_0 \left( \frac{1}{r = \infty} \right)
\]

where \( P_2 \) and \( P_1 \) are the first two coefficients of the second order polynomial, and \( P_0 \) is the extrapolated value. In a similar way other quantities e.g. \( Z_1 \) or \( tan(k_1 d) \) for \( r = \infty \) can be derived from the corresponding values of \( Z_1 \) or \( tan(k_1 d) \) for three measurement distances.

There are considerations for applying this extrapolation technique. It is only possible to extrapolate if near field effects vary sufficiently amongst the measurements at different heights. However, strong near field effects should be avoided because we use a plane wave model. For example, for \( r \geq 0.3 \, \text{m} \) and \( kr \geq 2 \) the minimum frequency is \( f_{\text{min}} = 364 \, \text{Hz} \). In addition, the measured impedance should not be too different from the plane wave impedance. Measurement inaccuracies may strongly affect the outcome if \( Z_{1(\infty)} \) is larger than about 2. Also, \( r \) should be larger than \( \sim 0.15 \, \text{m} \) in order to avoid influences from reflections against the loudspeaker. Furthermore, \( r \) should not be larger than \( \sim 0.5 \, \text{m} \) to ensure
that the loudspeaker signal sufficiently exceeds the (background) noise and that the direct sound wave is strong compared to reflections. Besides, a large \( r \) would be impracticable for a hand-held test set-up. The accuracy of the extrapolation improves if more measurements at different sound source heights are included.

5 Measurement results

In the following subsection, the values of the impedance measured above the sample are presented. Thereafter, the values of \( Z_1 \) and \( k_1 \) derived from the impedance above the sample are shown and discussed.

5.1 Measured impedance above the sample

Tests have been performed with and without a backplate as well as with two sample thicknesses. As example, figure 4 and 5 show the real part of the measured impedances near the sample for two sound source heights. The thick black line in these figures is the impedance \( Z_{m1d1} \) (the sample with a backplate and thickness \( d_1 \)). The thick grey line is \( Z_{m1d2} \) (the sample with a backplate and thickness \( d_2 \)). The thin black line is \( Z_{m2d1} \) (the sample without a backplate and thickness \( d_1 \)). And the thin grey line is \( Z_{m2d2} \) (the sample without a backplate and thickness \( d_2 \)). It can be seen that the values vary for different source heights because, the measurements are affected by near field effects differently, especially at low frequencies.

5.2 Obtained \( Z_1 \) and \( k_1 \)

Two sample configurations are required in order to find a solution. As was mentioned before, there are six possible solutions. The values of \( Z_1 \) and \( \tan(k_1d) \) that were found for all distances are then extrapolated to \( r = \infty \) using equation 4. The results obtained are shown in figure 6 to 9. It can be seen that the results from most combinations are similar. The lines in these figures correspond to the six combinations of pairs of measured impedances, see section 4.3 and Appendix B.
To obtain $k_1$ the arctangent is involved; for example for solution 1 $Z_{m1d1} = \frac{-iZ_1}{\tan(k_1d)}$ results in

$$k_1 = \frac{1}{2} \arctan(\frac{-iZ_1}{Z_{m1d1}}).$$

There are many possible solutions of this function because the tangent function is periodic, i.e. if a certain value of $k_1d$ fulfills equation $\pi$ a value of $k_1d + \pi$ also fulfills this equation. Therefore, $\pm \pi$ or a multiple of that has been added to $k_1d$ until $\Re(k_1d)$ deviates less than $\pi$ from the corresponding value in air.

### 5.2.1 Low frequency limitations

At low frequencies, the outcome of the plane wave extrapolation procedure may be incorrect because of several reasons. If $kr$ is too small the extrapolation procedure might be failing because the deviations from a situation with plane waves are large, and because the assumptions of the sound field model might be violated. In addition, the signal-to-noise ratio decreases due to the low loudspeaker signal. Furthermore, there are reflections during the tests, which mostly affect low frequencies where the modal density is low. Finally, there may be errors if the size of the sample is very small compared to the wavelength.

The error is significant when two configurations are similar. This is for example the case at low frequencies for the first solution ($Z_{m1,d1} \& Z_{m1,d2}$) and the last solution ($Z_{m2,d1} \& Z_{m2,d2}$) because the damping in the material is low and because wavelength is long compared to the difference in sample thickness. The results of these solutions have therefore been omitted below $\sim 350$ Hz. In addition, the obtained values of the imaginary part of the impedance seem to vary strongly below 700 Hz.

### 5.2.2 High frequency limitations

The impedances of two configuration can also be similar at high frequencies due to the high attenuation of the samples. In that case, measurement inaccuracies become dominant. Such deviations are found above 3.2 kHz with the last solution, and these results have therefore been omitted from figure 8 and 9.

In addition, there are strong fluctuations around 4.6 kHz for the real and imaginary part of the wavenumber for solution 5 (figure 8 and 9). The cause for these discrepancies is investigated further. Figure 10 shows $Z_{m1d2}$ and $Z_{m2d2}$ for $r=0.60$ m, and the value of $Z_1$ obtained from these impedances using solution 5 (see appendix B). In this graph, the real parts of these impedances are the three upper lines with values above 240 $\frac{\rho_0 c}{m}$; the imaginary parts are the three lower lines with values below 20 $\frac{\rho_0 c}{m}$.
Although $Z_{m1d2}$ and $Z_{m2d2}$ are similar at several frequencies, only around 4.6 kHz the real and imaginary parts of these impedances are similar. Furthermore, the real part approaches the characteristic impedance in air around this frequency, while the imaginary part is small. Consequently, $Z_{m1d2} \approx Z_{m2d2} \approx Z_1 = \rho c$. Equation 2 for a sample without a backplate now becomes:

$$Z_{\text{nobackplate}} = Z_1 \frac{iZ_1 \tan(k_1 d) + \rho c}{\tan(k_1 d) \rho c + Z_1} \approx Z_1 \approx \rho c$$  (5)

Hence, the measured impedance $Z_{m2d2}$ is independent of $k_1$. Equation 1 for a sample with a backplate can be rewritten to:

$$\tan(2k_1 d) = \frac{-iZ_1}{Z_{m1d2}} \approx -i$$  (6)

The wavenumber can be obtained by taking the arctangent of equation 6 and dividing by the sample thickness. Similar to the singularity in the tangent function (i.e. $\tan\left(\frac{1}{2} \pi\right) = \pm \infty$), a singularity exists in the arctangent function for $\tan(\pi) = 0 - i\infty$. Therefore, deviations occur when the measured $\tan(2k_1 d)$ approaches $-i$. Figure 11 shows the real and imaginary parts of $\tan(2k_1 d)$ which are obtained using solution 5. It can be seen that $\Im(\tan(2k_1 d)) \approx -1$ at multiple frequencies, but that only around 4.6 kHz $\Re(\tan(2k_1 d)) \approx 0$. When $\Re(\tan(2k_1 d)) \neq 0$ and $\Im(\tan(2k_1 d)) = -1$ the singularity of the arctangent function is less pronounced; for example, $\tan(-i) = 0 - \infty i$, while $\tan(0.05 - i) = 0.80 - 1.80i$ and $\tan(0.1 - i) = 0.81 - 1.50i$.

5.3 Normalised $Z_1$ and $k_1$

Figure 12 shows the mean value and the standard deviation (indicated by the coloured area’s) of the six solutions that were plotted in figure 6 to figure 9. The real and imaginary parts of $Z_1$ are normalised with the characteristic impedance in air, and the real and imaginary parts of $k_1$ are normalised with the characteristic wavenumber in air. The trends of the measurement results shown in figure 12 are similar to the simulated values of figure 1 and appendix A.
6 Validation through simulation

In this section, the procedure to obtain $Z_1$ and $k_1$ is validated through simulation. An approximate spherical wave model is used that was introduced in [16]. The details of the model can be found in appendix C. The model involves the summation of the sound pressure and particle velocity contributions of the individual travelling waves. Some of these travelling waves are reflected directly at the top surface of the sample, some go through the sample for a number of times. The simulated impedance near the sample depends on the height of the sound source because a near field correction is applied for each successive travelling wave.

The input parameters for the simulations are obtained from the Delany & Bazley model using a sample thickness of 0.027 m (equal to that of the Flamex sample) and a flow resistivity of $\sigma = 20$ kPa/m$^2$. The valid frequency range of the Delany & Bazley model depends on the flow resistivity and is in our case 200 Hz to 20 kHz.

The impedances that would have been measured near the sample are calculated with the sound field model for $r = 0.15, 0.20, 0.25, 0.30, 0.35, 0.40$, and 0.60 m. Next, $Z_1$ and $k_1$ are calculated using these impedances and the plane wave solutions, see section 4.3 and appendix B. Finally, the values of $Z_1$ and $k_1$ at different heights are extrapolated to $r = \infty$, which are then compared to the input values of the Delany & Bazley model. Figure 13 to 16 show the real and imaginary parts of $Z_1$ and $k_1$ for the solution 1 which requires $Z_{m1,d1}$ and $Z_{m1,d2}$.
The real and imaginary parts of $Z_1$ and $k_1$ in figures 13 to 16 have the same trend as during the measurements. It is possible to extrapolate properly at middle and at high frequencies because discrepancies between the extrapolated values (thick dashed grey lines) and the input values of the Delany and Bazley model (thick black lines) are small. However, difficulties are experienced at low frequencies where there are strong near field effects. For simulations, smaller deviations are found for thicker samples with higher flow resistivities.

7 Conclusions

The surface impedance, reflection, and absorption of a sample can be calculated for different mounting conditions if its bulk properties are known. A method has been introduced here to measure the complex characteristic impedance $Z_1$ and the complex wavenumber $k_1$ of a sample which is not clamped on the sides. Tests have been carried out in a broad frequency range (i.e. 200 Hz to 7 kHz) with a sound source that emits spherical waves above a large homogeneous sample. Several measurements with different configurations are combined because a single measurement near the surface does not yield enough information. Tests were performed without a sample, near a sample with and without a backplate, with two sample thicknesses, and with seven sound source heights.

The approach to obtain the bulk properties from these tests involves a plane wave model and an extrapolation procedure. In order to obtain $Z_1$ and $k_1$ two sample configurations are combined and the results from several sound source heights were extrapolated to $r = \infty$. The accuracy of the extrapolation procedure deteriorates if its input values vary strongly and if these values do not increase or decrease gradually with $r$. Similar results were obtained from the measurements with six combinations of sample configurations for frequencies where the extrapolation procedure was valid. In addition, simulations were done using a spherical sound field model. Valid results were obtained with the extrapolation procedure at middle and high frequencies.

Future research might be aimed to further investigation of the particularities of the method, to validation of the results, and to optimization of the measurement routine. Also, it is not necessary to use as many measurements as we have used. In principle only three distances and two sample configurations are required. Furthermore, this method might be extended to an (hand-held) in situ method used in environments with reflections or background noise. Even the usage of multiple probe positions, different sample sizes and/or different sound field types might be considered.

References


Appendix A: Delany and Bazley model

The Delany & Bazley model states that the characteristic sample impedance $Z_1$ and complex wave number $k_1$ can be calculated by [12] pp. 20-23:

$$Z_c = \rho_0 c_0 \left( 1 + 0.0570X^{-0.754} - 0.0870iX^{-0.732} \right) \quad (7)$$

$$k = \frac{2\pi f}{c_0} \left( 1 + 0.0978X^{-0.700} - 0.189iX^{-0.595} \right) \quad (8)$$

where $X$ is a dimensionless parameter:

$$X = \frac{\rho_0 f}{\sigma} \quad (9)$$

These expressions are valid when:

$$0.01 < \frac{f}{\sigma} < 1 \quad (10)$$

Figure 17 to 20 show the characteristic impedance and complex wave number calculated by the Delany and Bazley model for several flow resistivity values. The curves in these figures are dashed at frequencies where the boundaries in equation 9 are exceeded.
Appendix B: Calculation of bulk properties with the plane wave model

Measurements have been performed for several configurations of the sample; with and without sample, with and without a backplate, and with two sample thicknesses (i.e. \( d1 \) and \( d2 \), where \( d2 = 2d1 \)). In a situation with plane waves, \( Z_1 \) and \( k_1 \) can be obtained by equation (11) and by using at least two sample mounting configurations. To solve \( Z_1 \), first \( \tan(k_1d) \) is eliminated from these equations. In this appendix, several solutions are presented for the six specific combinations of configurations.

Solution 1: With a backplate for two sample thicknesses:

\[
Z_1 = \sqrt{2Z_{m1,d1}Z_{m1,d2} - Z_{m1,d1}^2} \quad (11)
\]

Solution 2: With and without a backplate for sample thickness \( d1 \):

\[
Z_1 = \sqrt{Z_{m1,d1}Z_{m2,d1} + Z_{m2,d1} \cdot \rho c - Z_{m1,d1} \cdot \rho c} \quad (12)
\]

Solution 3: With a backplate for sample thickness \( d1 \) and without a backplate for sample thickness \( d2 \):

\[
Z_1 = \sqrt{\frac{2Z_{m1,d1}Z_{m2,d2} + 2Z_{m1,d1}^2Z_{m2,d2} \cdot \rho c - Z_{m1,d1}^2}{2Z_{m1,d1} \cdot \rho c - Z_{m2,d2} \cdot \rho c + 1}} \quad (13)
\]

Solution 4: With a backplate for sample thickness \( d2 \) and without a backplate for sample thickness \( d1 \):

\[
Z_1 = \sqrt{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}} \quad (14)
\]

where \( a = 1 \), \( b = \rho^2c^2 + Z_{m2,d1}^2 - 4Z_{m2,d1} \cdot \rho c + 2Z_{m1,d2} \cdot \rho c - 2Z_{m1,d2}Z_{m2,d1}, \) and \( c = Z_{m2,d1}^2 \cdot \rho^2c^2 - 2Z_{m1,d2}Z_{m2,d1} \cdot \rho^2c^2 + 2Z_{m1,d2}Z_{m2,d1} \cdot \rho c \).

Solution 5: With and without a backplate for sample thickness \( d2 \):

\[
Z_1 = \sqrt{Z_{m1,d1}Z_{m2,d2} + Z_{m2,d2} \cdot \rho c - Z_{m1,d1} \cdot \rho c} \quad (15)
\]

Solution 6: Without a backplate for two sample thicknesses:

\[
Z_1 = \sqrt{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}} \quad (16)
\]

where \( a = 2Z_{m2,d1} / \rho c - Z_{m2,d2} / \rho c - 1 \), \( b = \rho^2c^2 - Z_{m2,d1}^2 - 2Z_{m2,d1} \cdot \rho c + Z_{m2,d2} \cdot \rho c + 2Z_{m2,d1}Z_{m2,d2} - Z_{m2,d1}Z_{m2,d2} / \rho c, \) and \( c = Z_{m2,d1}^2Z_{m2,d2} \cdot \rho c + Z_{m2,d1}^2 \cdot \rho^2c^2 - 2Z_{m2,d1}Z_{m2,d2} \cdot \rho^2c^2 \).

Actually, there should have been a \( \pm \) sign before the main square root in equation (11) to (16) but this sign can be omitted because it is known that the real part of \( Z_1 \) is positive. Subsequently, \( k_1 \) can be obtained by the calculated \( Z_1 \) and, depending on the configurations used, by equation (1) or (2).

Appendix C: Spherical Reflection Model

A spherical model is used to calculate the pressures and velocities, and thus the impedance and intensities above and below the sample. A summation is used of travelling waves that are reflected at the top surface and go through the sample several times.

For plane waves this summation of travelling waves leads to simple algebraic expressions, as given in equation (1) and (2). However, for spherical waves it is not straightforward to find a simple algebraic expression. Therefore, we used a numerical procedure. The reason that simple algebraic expressions could not be found is that, for spherical waves, there is not only a \( \frac{1}{r^2} \) term in the sound pressure and particle velocity, but also a \( \frac{1}{\sqrt{r^2}} \) term in the particle velocity. Therefore, there is a dependence of distance to the various virtual sources corresponding to a travelling wave. For each successive travelling wave a near field correction is made. As input for the model the complex wavenumber and surface impedance are derived from the Delany & Bazley equations, see appendix A. Other, more sophisticated models, might be used as well.
Some sound will reflect directly at the top surface due to the impedance jump at the air-sample interface. A portion will also penetrate through the surface. Inside the sample there will be some degree of absorption. Depending on the medium behind the sample a portion of the sound will be reflected back towards the receiver. The part of the sound that will be transmitted out of the sample will attribute to the measured sound pressure and velocity. The reflected part might be absorbed, reflected against the back of the sample, and so forth... The result is an evanescent wave inside the sample of which the components that transmit through the top surface contribute to the pressure and velocity above the surface. The matter is illustrated in the next figure:

![Figure 21: Travelling waves in vertical direction.](image)

For a sound wave going from air towards the sample reflection $R_{01}$ and transmission $T_{01}$ are formulated as:

$$R_{01} = \frac{x - 1}{x + 1}$$  \hspace{1cm} (17)

$$T_{01} = e^{-\pi(k_0 - k_1)} \frac{2x}{x + 1}$$  \hspace{1cm} (18)

where $x$ is:

$$x = \frac{Z_1 (1 + \frac{i k_0 r}{k_1 r})}{Z_0 (1 + \frac{i k_0 r}{k_1 r})}$$  \hspace{1cm} (19)

$Z_1$, $Z_0$ and $k_1$, $k_0$ are the impedance and wavenumber of resp. the sample and air. For a sound wave going from the sample towards the air reflection $R_{10}$ and transmission $T_{10}$ are:

$$R_{10} = \frac{1 - x}{1 + x}$$  \hspace{1cm} (20)

$$T_{10} = e^{-\pi(k_0 - k_1)} \frac{2}{x + 1}$$  \hspace{1cm} (21)

For the impedance normalized to air ingoing pressure $p_1$ and velocity $u_1$ are:

$$p_1 = p_0 \frac{e^{-i k_0 r}}{r} \hspace{1cm} u_1 = p_1 \left(1 + \frac{1}{ik_0 r}\right)$$  \hspace{1cm} (22)

where $p_0$ is the amplitude related to the strength of the monopole. The $p_2$ and $u_2$ of the first reflection at the material surface are:

$$p_2 = p_1 R_{01} \hspace{1cm} u_2 = -u_1 R_{01}$$  \hspace{1cm} (23)

The contribution to pressure and velocity above the sample of a wave going $n$ times through the sample is:

$$p_{(4n+2)} = p_0 e^{-i k_0 r} r + 2nd \frac{T_{01} R_{12} R_{10}^{n-1} e^{-2nk_1 d}}{T_{10}}$$  \hspace{1cm} (24)

$$u_{(4n+2)} = u_{(4n+2)} \left(1 + \frac{1}{i k_0 (r + 2nd)}\right)$$  \hspace{1cm} (25)

where $R_{12}$ is the reflection coefficient at the backside of the sample which is equal to one if there is a backplate, and equal to $R_{10}$ if there is no backplate. The actual measured pressure $p_m$ and velocity $u_m$ above the sample is the summation of the ingoing wave, the initial reflection and all the contributions of the waves that went through the sample:

$$p_m = p_1 + p_2 + \sum_{n=1}^{\infty} p_{(4n+2)}$$  \hspace{1cm} (26)

$$u_m = u_1 + u_2 + \sum_{n=1}^{\infty} u_{(4n+2)}$$  \hspace{1cm} (27)

Behind the sample there are only sound waves that have travelled through the material. The measured pressure and velocity are the summation of:

$$p_{t(4n)} = p_0 e^{-i k_0 r} r + 2nd - d \frac{T_{01} \left(R_{10}^{n-1}\right)^2 e^{-i k_1 d(1+2n)}}{T_{10}}$$  \hspace{1cm} (28)

$$u_{t(4n)} = -p_{t(4n)} \left(1 + \frac{1}{i k_0 (r + 2nd - d)}\right)$$  \hspace{1cm} (29)

With simulations it was found that all sound waves that are reflected more than 6 times between the backside of the sample and the top surface contribute little to the measured sound pressure and particle velocity above the sample. For the simulations as they are done in this paper twenty travelling waves are included for safety.

An exact solution cannot easily be obtained for spherical waves because a different near field correction is applied for each sound wave that travels one or multiple times through the material. However, for plane waves an exact solution can be found by treating each successive wave as part of an infinite series. In case $h$ goes to infinity $x$ can simply be replaced by the normalized characteristic material impedance $Z_{cn} = \frac{Z_0}{Z_1}$, and equation 26 and 27 converge with respectively equation 11 and 12.