An Intensity Method for Measuring Absorption Properties \textit{in situ}

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Summary

The well-known Kundt’s tube and reverberant room method are often used for measurement of acoustic absorption properties of samples under laboratory conditions. Several \textit{in situ} measurement methods exist, but most of them are limited in frequency range, require large samples and/or are vulnerable to background noise or reflections.

The PU \textit{in situ} impedance method [1, 2] has been used successfully on relatively small samples (>0.1 m$^2$) in a broad frequency range (300 Hz – 10 kHz) under reverberant conditions (e.g. a car interior or a concert hall), see e.g. [3, 4, 5, 6, 7]. The small source-sample and probe-sample distance are the main reasons for the relative small sample size requirement and the low influence to background noise and reflections.

However, in some cases the procedure shows artefacts because all the reflection at the top of the sample is considered, not taking into account wave propagation in the material. In this research the principle of measuring intensity instead of impedance is investigated. To eliminate near field effects an extrapolation technique is introduced that combines several measurements. The result is a technique to measure the absorption coefficient without knowledge of the material. The methods are examined theoretically and verified with experiments.

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1. Introduction

The PU \textit{in situ} impedance method makes use of a pressure-velocity probe that is positioned close to the material and of a point source at some distance from the sample. Quantities such as local impedance and intensity can be measured directly.

A small portable device is often used that combines a PU probe with a sound source which is very similar to a true point source, see Figure 1, [4, 6]. The source-probe distance and the probe-surface distance are kept small which reduces the required sample size and the influence of reflections and/or background noise. Because a fixed configuration is used for the sample measurement and the calibration, positioning and orientation problems of the probe relative to the sound source are avoided.

A drawback of this small device is that the sensors are in the near field of the source and the reflecting surface. For this reason a simple plane wave approximation for interpreting experimental results is not adequate and a correction for the spherical sound field has to be made.

Many authors (e.g. [1, 2, 3, 4, 5, 6, 7, 8]) have addressed this issue but most often models are proposed that assume a reflection only at the top surface of the sample, as if the sample would be infinitely thin. This assumption is valid if there truly is one surface of reflection; examples are a rigid impervious plate (100% reflection), or a perfect anechoic termination (100% absorption).

In reality there is an initial reflection at the top surface of most samples because of the different characteristic impedance than air. But also a portion of sound penetrates through the surface and can travel numerous times between the top surface and the back of the sample. Thus a spherical standing wave will occur inside the sample, which needs to be corrected for.

Here another approach is used based on active intensity, which is the real part of the cross-spectrum of pressure and velocity. Whereas the absorption coefficient is defined as the ratio absorbed to incoming sound intensity, a measurement based on intensity seems logical. However in practice difficulties are experienced [9]. Measurements with traditional p-p intensity probes are often hampered by high pressure-intensity index ratios [10, 11, 12]. Also techniques combining intensity and sound energy have been attempted [13]. But none of the these methods seem to compensate for near field effects in case of a point source above a material.

The procedure to interpret the measurements in terms of the real part of intensity seems to be simple and straightforward, however, near field effects in the spherical waves complicate the matter and have to be taken into account.

An extrapolation technique which combines several intensity measurements with various sound source distances is introduced. Both the measurement procedure and the ex-
Acoustic absorbing materials can be installed in several ways and this will impact the final absorption coefficient measured by the setup. For this reason samples with backplate (typical conditions for e.g. cars), and without backplate (typical for e.g. office walls) are analyzed.

2. Description of the Experiments

The sound source from Figure 1 has been used to measure three different arrangements, see Figure 2. Here $I_0$ corresponds to intensity from the sound source at distance $h$, and is in fact used as calibration of the sensors in the free field. $I_1$ is intensity at a distance $h_s$ above the sample with rigid fully reflecting backplate. $I_2$ and $I_3$ are measured in front, respectively behind a sample without backplate.

The sound field above a sample can be highly reactive, especially if the reflection is high. This affects the intensity measurement. For the sample with backplate sound source distance $h$ has been varied from 25 mm to 625 mm in 25 mm steps. Distance $h_s$ and $h_t$ were 5 mm. Three materials are measured; an open melamine foam called Flamex, a dense PU foam and a linoleum floor (sample thickness resp. 27 mm, 28 mm and 2.5 mm). In all cases sample and backplate could be considered large ($>1.5 \text{ m}^2$) relative to the source-probe-sample distance.

3. Physical Method

3.1. Definitions

Absorption coefficient $\alpha$ is defined as the ratio of absorbed to incident sound intensity. In the situation of $I_1$, the sample with backplate, it is assumed there is no transmission through the material because of the impenetrable backplate thus $\alpha$ can be calculated [14],

$$\alpha = \frac{\Re(I_{\text{incident}} - I_{\text{reflected}})}{\Re(I_{\text{incident}})} = \frac{\Re(I_1)}{\Re(I_2)}. \quad (1)$$

Intensity $I_1$ is the sum of incident- and reflected intensity normal to the sample surface (the latter with a negative sign because it is directed in the opposite direction of $I_0$).

For a sample without a backplate, the transmission through the sample also has to be taken into account. With intensity $I_2$ in front of the sample, and intensity $I_3$ behind the sample, absorption $\alpha$ and intensity transmission coefficient $T_1$ can be calculated,

$$\alpha = \frac{\Re(I_{\text{incident}} - I_{\text{reflected}}) - \Re(I_{\text{transmitted}})}{\Re(I_{\text{incident}})} = \frac{\Re(I_2) - \Re(I_3)(\frac{h + d + 2h_s}{h})^2}{\Re(I_0)}, \quad (2)$$

$$T_1 = \frac{\Re(I_{\text{transmitted}})}{\Re(I_{\text{incident}})} = \frac{\Re(I_3)(\frac{h + d + 2h_s}{h})^2}{\Re(I_0)}. \quad (3)$$

In these equations a correction for the extra distance $d + 2h_s$ of $I_3$ from the source is made.

Whereas absorption is the ratio of absorbed to ingoing sound intensities usually transmission loss defined in decibels is used instead of the intensity transmission coefficient; $TL = 10\log(T_1)$. 

Figure 1. Portable in situ measurement set-up.

Figure 2. Three measurement arrangements.
3.2. Calibration

The measurement at position \( I_0 \) can also be used to calibrate the intensity probe if the free field impedance is known. This sound source with a loudspeaker inside a spherical housing is chosen because the impedance in front of the source can be described and because it has a higher radiation efficiency compared to a point source. In [15] this type of sound source was modelled as a moving piston on a rigid sphere, and a description of impedance in front of the sphere was given.

With the existing source- and setup dimensions the phase difference with that of a point source is small; maximum \( \pm 1 \) degrees in a 10 Hz–10 kHz band (\( h > 0.1 \) m). It is therefore in our case it is not necessary to use the piston-on-a-sphere description. Free field impedance \( Z_{\text{cal}} \) at distance \( h \) of a point source and intensity \( I_{\text{cal}} \) are [15]

\[
Z = \frac{p}{u} = \frac{i k_0 h}{k_0 h + 1} \rho c
\]

(4)

\[
I_{\text{cal}} = \frac{p^2}{Z_{\text{cal}}}
\]

(5)

Where \( p \) and \( u \) are the measured pressure and velocity, \( k_0 \) is the wavenumber in air and \( ^* \) denotes the complex conjugate. The influence of constants air density \( \rho \) and sound velocity in air \( c \) are not considered; the normalized impedance is used instead for the calibration and the sample measurements.

Now the impedance during measurement \( I_0 \) is known the procedure to correct for the complex sensitivities of both sensors is quite easy to implement. True intensity \( I_{\text{true}} \) is the measured intensity \( I_{\text{meas}} \) (or \( I_{\text{int}}, I_{\text{2mr}}, I_{\text{lin}} \)) times a correction \( A_{\text{corr}} \) for amplitude and \( \Phi_{\text{corr}} \) for phase:

\[
I_{\text{true}} = |I_m| e^{i \Phi_{\text{cor}}} A_{\text{corr}} e^{-i \Phi_{\text{cor}}}
\]

(6)

Actual amplitude \( A_{\text{corr}} \) appears on both sides of the fraction \( I_1/I_0 \) (or \( I_2/I_0 \) or \( I_3/I_0 \)) and can therefore be omitted. The phase correction can be calculated from measurement \( I_{\text{meas}} \) and equation (4) and (5): \( \Phi_{\text{corr}} = \Phi_{\text{meas}} - \Phi_{\text{cal}} \).

The corrected \( I_{\text{true}} \) is used for all measured and simulated values in this paper.

3.3. Considerations

Because impedance is the ratio of pressure and velocity it is not affected by a changing source strength. A disadvantage of a method based on intensity is that the source strengths between \( I_0 \) and \( I_1 \) could differ, especially if \( I_1 \) is measured in conditions with a few strong reflections. However, because of the short probe-source distance this effect is negligible.

During our experiments the probe was only placed at a single position on the surface with the sound source at normal incidence. Actually the real part of intensity should be integrated over a closed surface in order to avoid local deviations and to obtain the absorption coefficient integrated over all angles of incidence. This means that a scan or multiple points should be measured of a larger surface area for each distance \( h \) while keeping the sound source at a fixed position and the probe oriented towards the surface.

Figure 3. Phase between P and U in the free field and near three different samples (\( h = 0.3 \) m).

4. Sound Field Reactivity

Main sources of error of traditional p-p intensity probes are related to finite difference approximation, scattering and diffraction, microphone size and instrumentation phase mismatch. Because of the latter, pressure should typically not exceed intensity with more than 10 dB to 20 dB, depending on frequency.

The error of PU probes on the other hand is mostly related to the reactivity of the sound field. In [16] it was mentioned that in practical situations the error would increase significantly if the imaginary intensity exceeds the real part with more than 5 dB. This corresponds to a phase between pressure and velocity of 72 degrees. This 5 dB or 72 degrees boundary is also used as limit here for the intensity measurements to be valid. In Figure 3 the measured phase after correction with \( \Phi_{\text{corr}} \) is shown for the following cases (samples with backplate):

- Free field conditions without a sample, using equation (5) (black line).
- Near Flamex sample (grey line).
- Near PU foam sample (black dashed line).
- Near Linoleum sample (grey dashed line).

As can be seen in Figure 3 the reactivity is not so high during the calibration measurement \( I_0 \) (black line) and the phase remains between \( \pm 72 \) degrees. Near the samples the reactivity can be much higher. In such situations the error of p-u intensity measurement increases. In Figure 4 the phase of the separate incoming wave \( (I_0) \) and the reflected waves \( (I_{\text{sample}}-I_0) \) are shown.

5. Measurement at Several Distances

From the measurements of pressure and particle velocity the real part of intensities \( I_1, I_2 \) and \( I_3 \) are calculated and divided by the real part of intensity \( I_0 \). A quantity which should be related to the absorption of the sample is \( AI_1 = \frac{R(I_1)}{R(I_0)}, AI_2 = \frac{R(I_2)}{R(I_0)}, AI_3 = \frac{R(I_3)}{R(I_0)} \).
Intensity is measured in three arrangements (as shown in Figure 2) and the reflection-, absorption- and transmission coefficients are calculated. To remove some of the parasitic reflections from the room the measurements were smoothed using a moving average in the frequency domain [4]. Various distances of \( h \) have been measured.

5.1. With Backplate (\( I_0 \) and \( I_1 \))

Figure 5 shows \( AI_1 \) of the Flamex sample with backplate. Dotted parts of the lines are frequencies where the phase exceeded \( \pm 72 \) degrees. Reactivity is mainly high at low frequencies and increases for larger sound source distances.

There are strong near field effects, especially for small distances of \( h \), and invalid results are obtained for 'absorption' \( AI_1 \) (larger than one). Therefore a correction for these near field effects needs to be applied.

5.2. Without Backplate (\( I_0, I_2 \) and \( I_3 \))

Next, intensity transmission and -absorption of the same sample without backplate are calculated using measurement \( I_2 \) in front and \( I_3 \) behind the sample. Not surprisingly the absorption is different from the situation with plate because the boundary conditions of the sample have changed. The results are shown in Figure 6 to 8.

Again strong near field effects are measured, especially in \( AI_2 \). Apart from that \( AI_2 = AI_3 \), which is related to absorption, does not tend to go to zero at low frequencies as was the case for the sample with backplate.

6. Physical Interpretation of the Results

In this section we try to explain the experimental results from the previous chapter using simple models incorporating travelling waves. Let’s consider Figure 9, where the intensity probe is located at \( z = 0 \), detecting intensity in the z direction. From sound source \( S \) at a distance \( h \) above the sample the direct wave 1 travels towards the sample.
is partly transmitted, transforming into wave 4, and partly reflect becoming wave 5. The degree of transmission or reflection depends on the impedance of the medium behind the sample. In case there is a backplate there will be no transmission and wave 4 will thus be zero.

At the top of the sample wave 5 will be reflected and transmitted. The transmitted wave 6 will contribute to the pressure and velocity above the sample. The reflected wave 7 will travel back into the sample and will be reflected and transmitted against the top & back surface numerous times (becoming wave 8, 9, 10,...etc). Waves 3, 5, 7...etc are damped due to absorption in the sample.

6.1. $A_{I_2}$: Reflected- exceeding incoming intensity

First the case with backplate is studied, see Figure 10. As there is no transmission through the back of the sample wave 4, 8, 12,...etc are zero. Wave 2 which is the first reflection at the top of the sample can be represented as mirror source $M_2$, located at $z = -h$. Also wave 6, 10, 14,...etc contribute to intensity above the sample. When sound has travelled through the sample twice $M_6$ (mirror source of wave 6) will be located at $z = -(h + 2d)$. $M_{10}$ corresponds to the mirror source of wave 10 at $z = -(h + 4d)$. Each successive mirror source will be located further from the sensor at multiples of 2d.

Waves 1, 2, 6, 10,... etc. contribute to the sensor signal. Their individual intensity contributions $I_5$, $I_{M_2}$, $I_{M_6}$,...etc are now considered separately. The contribution of wave 2, relative to wave 1, is determined only by the degree of reflection of the sample and is therefore independent of distance $h$. When mirror source $M_2$ and direct source $S$ would have equal strength the net intensity at the probe position would be zero. However, when mirror source $M_6$ would have equal strength as $S$ instead of $M_2$ this would not be the case. $M_6$ is at a distance $h+2d$, while $S$ is at a distance $h$. If the source strengths of wave 1 and 6 are equal intensity will not vanish and depends on $h$.

Fig. 9. Illustration of individual travelling waves above a sample in vertical direction.

Fig. 10. Travelling waves for a sample with backplate.

Including only wave 1, 2 and 6 the pressure and the velocity at the probe position are written

$$p = \frac{1}{h} + S_2 \frac{1}{h} + S_6 \frac{e^{-ik2d}}{h+2d}$$  \hspace{1cm} (7)

$$u = \frac{1}{hc} \left(1 + \frac{1}{ikh}ight) + S_2 \frac{1}{h} \left(1 + \frac{1}{ikh}ight) + S_6 \frac{e^{-ik2d}}{h+2d} \left(1 + \frac{1}{ikh(h+2d)}\right)$$ \hspace{1cm} (8)

where $S_2$ and $S_6$ are the source strength of resp. wave 2 and 6, with the condition; $S_2 + S_6 < 1$. After some algebra (see appendix A) the ratio of intensities with ($I_1$) and without sample ($I_0$) becomes

$$\frac{R(I_1)}{R(I_0)} = 1 - S_2^2 - \frac{S_6h}{h+2d} - \frac{2hS_6S_2\cos(kd)}{h+2d} + S_6\sin(2kd) \frac{b(1+S_2)+(h+2d)(1-S_2)}{k(h+2d)^2}$$ \hspace{1cm} (9)

In Figure 11 this ratio of intensities is plotted for various distances of $h$. Here is assumed that wavenumber $k$ of the material is equal to that of air, thus that there is no absorption inside the sample. In this example $S_2$ has a value of 0.2, and $S_6$ is 0.8 ($d = 0.027$ m). It appears that
the resulting intensity, relative to \( I_0 \), can be larger than one (and this does not correspond to negative absorption!). This extra contribution to the real part of intensity is related to the multiplication of the imaginary part in pressure \( e^{-ikd} \) with the imaginary part in particle velocity \( (1/\omega h) \), which results in a positive contribution to the real part of intensity. A similar calculation can be done for wave 10, 14, 18, etc. This simple model is an example that shows that \( I_1 \) can be larger than \( I_0 \).

### 6.2. \( AI_2 \): High Values of for a sample without backplate

At first sight high values of \( AI_2 = AI_3 \) without backplate at larger source-probe distances \( h \) might be a surprise. Such high ‘absorption’ values at low frequencies seemed counterintuitive for a thin sample without backplate. These high values are caused by the interference of secondary reflections that went through the material.

Let us consider Figure 12 where several successive travelling waves are shown for a sample without backplate. At each air sample boundary there will be a reflection. Expressions for the reflection and transmission at an impedance boundary for medium A and B that can be found in most textbooks (e.g.
\[ R = \frac{Z_A - Z_B}{Z_A + Z_B} \quad T = \frac{2Z_A}{Z_A + Z_B} \] (10)

Wave 2 is the portion of wave 1 that is reflected directly at the top surface. Typically the impedance of the sample is higher than the air, thus for a wave going from the air towards the sample the reflection is positive; \( R = Z_{sample} - Z_{air}/Z_{sample} + Z_{air} \geq 0 \). Since wave 2 travels in opposite direction from wave 1 it will decrease the intensity above the surface. Wave 1 will also partially penetrate through the top surface. This transmitted wave is dampened to some degree and then partially reflected against the bottom of the sample because of the impedance transition. The crux is that for a sound wave travelling from inside the sample towards the air there is a negative reflection;
that depend on $h$ in forms like $1/h, 1/(h+2d), 1/h^2$. The total intensity contains a mixture of these terms and therefore we use $B/h^m + a$. The increase of $1/I_0$, such as in Figure 11, appears mainly at low frequencies where damping of many materials is weak, thus mirror sources $M6, M10...$ etc become more important while there are strong near field effects.

7.2. Optimization Method

In order to reduce noise from individual measurements $AI$ at 25 positions is combined. Sound source distance $h$ was varied from 25 mm to 625 mm in 25 mm steps. The optimization routine to calculate absorption consists of the following steps:

1. Values of $AI$ were the phase exceeds $\pm$ 72 degrees are ignored to avoid inaccuracies in the intensity calculation.

2. $AI_1$ and $AI_2$ tend to decrease for larger distances of $h$ (see e.g. Figure 5 and 6). Therefore values of $AI_1$ or $AI_2$ are ignored if they are smaller than those of a larger distance. The opposite is the case for $AI_3$; its values increase for larger values of $h$ (see e.g. Figure 7). Values of $AI_3$ are ignored if they exceed those of a larger distance.

3. A least mean square (LMS) fit of equation (11) is done for all possible pairs of three measurement distances. Typical starting values are $B = 0.05, m = 1$ and $a = 0.5$. In our case there were 25 distances, so there were many calculations.

4. The constant $m$ is most often a value near one. To remove outliers only LMS solutions where $m$ is between 0 and 2 are used.

5. If input values of the $AI$ pair are almost equal, the near field effects are small and the values are close to the actual absorption $a$. Small measurement inaccuracies could cause unpredictable outcomes of equation (11). The following formulas are used to condition this equation: $X$ is the exponential relation of the standard deviation of the input times a constant $c$; $X = 1/e^{a h(AI)c}$ ($c = 15$ in our case). The LMS solution and th of $AI$ are then combined: $a_{estimate} = (1 - X) \cdot a_{LMS} + X \cdot mean(AI)$.

6. Finally the median value of all valid $a_{estimate}$ solutions is taken.

7.3. Using a Limited Amount of Distances

It is not very practical to combine a large number of measurements. Here the quality of the estimated absorption is studied if less distances are used. For $h > 0.4$ m the reactivity is high in a broad band for typical samples because of the high reflection at low frequencies. Measurements at these sound source positions generally contribute little to the final solution. Small distances of $h$ should be avoided because of reflections against the sound source. Also $AI$ values should be combined where $h$ increases sufficiently.

Figure 15 shows an example of 2/3/4/5 measurement distances combined for the Flamex sample with backplate.
As expected, the estimation improves if more measurement positions are included, but even with two positions results are quite acceptable. In the latter case \( m \) would be 1 and instead of using the LMS method also the following direct solution can be used,

\[
AI_{h \to \infty} = \frac{AI_{h1} - AI_{h2}}{h_1 - h_2},
\]

where \( AI_{h1} \) and \( AI_{h2} \) are the \( AI \) (or \( AI_2 - AI_3 \)) at two source probe distances \( h_1 \) and \( h_2 \).

8. Extrapolation Results and Comparison with the Kundt’s tube

This extrapolation principle to calculate plane wave absorption has been applied to all three materials with a backplate. For comparison absorption is also measured with the \textit{in situ} mirror source method \([8, 1, 2, 3, 7, 4, 6]\), using a 27 cm probe-source distance, and with a Kundt’s tube. Results of the three methods are shown in Figure 16 and 17.

Above 1.5 kHz the reactivity was too high for the linoleum sample because of its high reflection coefficient, and there were not enough valid least mean square solutions. However at high frequencies traditional impedance based models can be used if near field effects are small.

Results of the Kundt’s tube an the intensity based extrapolation method are in good agreement. Similar absorption values are also found with the mirror source method for the Linoleum sample and above 400 Hz for the Flamex and PU foam sample. Below 400 Hz deviations are found (values below zero) because spherical waves inside the sample are not taken into account with the latter model.

Also far field intensity reflection, absorption and transmission coefficients are calculated from the measured intensities in front- and behind the Flamex sample without backplate (Figure 18).
9. Validation with Simulations

Spherical waves above—and inside the sample are included for the models that are used for figures in this chapter. Details of the models are explained in appendix B. They make use of a summation of travelling waves that are reflected at the top surface and go through the sample for a number of times. A near field correction is applied for each successive travelling wave. Typically sound waves that are reflected more than 6 times between the backplate and the top surface contribute little to the measured intensity above the material. Here for safety 20 waves are included.

The input parameters are obtained from the well-known model of Delany & Bazley for fibrous anisotropic materials [18, p. 25]. A flow resistivity that is typical for sound absorbing materials ($\sigma = 25000$ rayl/m) and a sample thickness of 0.027 mm (equal to that of the Flamex sample) are used here. The valid frequency range of the Delany & Bazley model depends on the flow resistivity and is in our case 250 Hz to 25000 Hz.

9.1. Simulated phase in front of the sample

The following two figures show the simulated phase in front of the sample, with- and without backplate. The simulated values of phase behind the sample are not shown here but they are similar to the free field phase; $\varphi(I_0) \approx \varphi(I_1)$. The same trend as during the measurement is shown for the sample with backplate. The phase becomes more negative for larger distances $h$. The field in front of the sample without backplate is much less reactive and the phase is closer to zero.

9.2. Simulated $AI_1$, $AI_2$ and $AI_3$

Figure 21 and 22 show the values of $AI_1$, in front of the sample with backplate, and $AI_2 - AI_3$ without backplate. Like is shown in the measurements results, an overestimation of $AI$ is made for smaller distances of $h$.

9.3. Calculated Absorption

These simulated values of $AI$ are then used to calculate the plane wave absorption using the extrapolation model. Ab-
Three samples with- and without backplate have been investigated. A technique has been introduced to extrapolate the plane wave absorption coefficient from the measured intensity above the surface using two or more source probe distances. The estimated absorption values are in good agreement with the measurements from a Kundt’s tube (see Figure 16, 17, 18). In addition the extrapolation method has been validated using simulations.

**Appendix**

**A1. Simple Model for S and MS6**

In this appendix intensity at the probe position is derived using a simple model. Here only direct source $S$, $M2$ and $M6$ are considered. $M2$ represents the initial reflection at the top of the material. $M6$ is the mirror source from wave 6; the first wave that is reflected at the back of the sample.

\[
p = \frac{1}{h} \left( S_1 \frac{1}{h} + S_2 e^{-ik2d} + S_6 \frac{e^{-ik2d}}{h+2d} \right) \tag{A1}
\]

Then intensity above the sample is

\[
\frac{\Re(I_1)}{pc} = \text{re} \left[ \left( \frac{1}{h} + S_6 \frac{e^{-ik2d}}{h+2d} \right) \cdot \left( S_2 - \frac{1}{h} \left( 1 - \frac{1}{ikh} \right) \right) \right] \right] \tag{A3a}
\]

\[
+ \text{re} \left[ S_6(1 + S_2)e^{ik2d} - S_6(1 - S_2)e^{-ik2d} \right] \left( h(h+2d) \right) \right] \tag{A3b}
\]

\[
+ \text{re} \left[ S_6(1 + S_2)e^{ik2d} - S_6(1 - S_2)e^{-ik2d} \right] \left( h(h+2d) \cdot kh \right) \right] \tag{A3c}
\]

10. **Conclusion**

A new interpretation of PU *in situ* measurements has been proposed. A spherical sound source and a PU probe are placed near the surface of an absorbing material. Instead of calculating the absorption coefficient from the near field impedance as is already proposed many times, the real part of intensity is used from which the far field radiation is obtained.

The sound pattern with a point source near the surface is quite complex and relative strong near field effects occur. Also the damping in the material cannot be described in detail with a single measurement near the surface.

Calibration of the sensors is simple; the responses are measured in the free field and a correction for the phase of the sound field is applied.
The ratio of intensity above the sample $I_1$ to ingoing intensity $I_0$ then finally becomes:

$$
\frac{R(I_1)}{R(I_0)} = 1 - S^2 - \left( \frac{S6h}{h + 2d} \right)^2 - \frac{2h6S2 \cos(kd)}{h + 2d} \frac{h(1 + S2) + (h + 2d)(1 - S2)}{k(h + 2d)^2}.
$$

(A4)

A2. Spherical Reflection Model

A spherical model is used to calculate the pressures and velocities, and thus the impedance and intensity, above and behind the sample. A summation is used of travelling waves that are reflected at the top surface and go through the sample several times. For each successive travelling wave a near field correction is made. As input for the model the complex wavenumber and surface impedance are derived from the Delany & Bazley equations. Other, more sophisticated models, might be used as well of course.

A2.1. Delany & Bazley Equations

The Delany & Bazley model states that characteristic sample impedance $Z_c$ and complex wave number $k$ are calculated by [18]

$$
Z_c = \rho_0 c_0 \left( 1 + 0.057X^{-7.54} - 0.087iX^{-7.32} \right),
$$

(A5)

$$
k = \frac{2\pi f}{c_0} \left( 1 + 0.978X^{-7} - 1.89iX^{-5.95} \right),
$$

(A6)

where $X$ is a dimensionless parameter,

$$
X = \frac{\rho_0 f}{\sigma}.
$$

(A7)

Boundaries that are mentioned for the validity of these expressions are

$$
0.01 < \frac{f}{\sigma} < 1
$$

(A8)

A2.2. Basic Assumptions

Some sound will reflect directly at the top surface because of the impedance jump at the air-sample interface. A portion will also penetrate through the surface. Inside the sample there will be some degree of absorption. Depending on the medium behind the sample a portion of the sound will be reflected back towards the receiver. The part of the sound that will be transmitted out of the sample will attribute to the measured sound pressure and velocity. The reflected part might be absorbed, reflected against the back of the sample, and so forth. The result is an evanescent standing wave inside the sample of which the components that transmit through the top surface contribute to the pressure and velocity above the surface. The matter is illustrated in the FigureA1.

For a sound wave going from air towards the sample reflection $R_{01}$ and transmission $T_{01}$ are formulated as

$$
R_{01} = \frac{x - 1}{x + 2},
$$

(A9)

$$
T_{01} = e^{-ik(k_0 - k_1)} \frac{2x}{x + 1},
$$

(A10)

where $x$ is

$$
x = \frac{Z_1 \left( 1 + \frac{1}{ik_0h} \right)}{Z_0 \left( 1 + \frac{1}{ik_0h} \right)}.
$$

(A11)

$Z_1$, $Z_0$ and $k_1$, $k_0$ are the impedance and wavenumber of resp. the sample and air. For a sound wave going from the sample towards the air reflection $R_{10}$ and transmission $T_{10}$ are

$$
R_{10} = \frac{1 - x}{1 + x},
$$

(A12)

$$
T_{10} = e^{-ik(k_0 - k_1)} \frac{2x}{x + 1}.
$$

(A13)

For the impedance normalized to air ingoing pressure $p_1$ and velocity $u_1$ are

$$
p_1 = \frac{e^{-ik_0h}}{h}, \quad u_1 = \frac{1}{ik_0h} \left( 1 + \frac{1}{ik_0h} \right).
$$

(A14)

The $p_2$ and $u_2$ of the $1^{st}$ reflection at the material surface are

$$
p_2 = p_1 R_{01}, \quad u_2 = -u_1 R_{01}.
$$

(A15)

A2.3. P&U above a Sample with Backplate

The contribution to pressure and velocity above the sample of a wave that is going $n$ times through the sample is

$$
p_{(4n+2)} = \frac{e^{-ik_0h}}{h + 2nd} T_{01} R_{10}^{n-1} e^{-ik_0h} d T_{01}.
$$

(A16)

$$
u_{(4n+2)} = p_{(4n+2)} \left( 1 + \frac{1}{ik_0(h + 2nd)} \right).
$$

(A17)

The actual measured pressure $p_m$ and velocity $u_m$ above the sample is the summation of the ingoing wave, the initial
reflection and all the contributions of the waves that went through the sample
\[
p_m = p_1 + p_2 + \sum_{n=1}^{\infty} p_{i(4n+2)}, \quad \text{(A18)}
\]
\[
u_m = \nu_1 + \nu_2 + \sum_{n=1}^{\infty} \nu_{i(4n+2)}. \quad \text{(A19)}
\]

A2.4. P&U above a sample without backplate
Without backplate sound waves can also exit via the backside of the material,
\[
p_{i(4n+2)} = \frac{e^{-i k h}}{h + 2n d} T_{i0}^n \rho_{10}^n R_{10}^{n-1} e^{-i k i d T_{10}}. \quad \text{(A20)}
\]
\[
u_{i(4n+2)} = -p_{i(4n+2)} \left( 1 + \frac{1}{i k_0 (h + 2n d)} \right). \quad \text{(A21)}
\]

B.5. P&U Behind a Sample without backplate
Behind the sample there are only sound waves that have travelled through the material. The measured pressure and velocity are the summation of
\[
p_{i n} = \frac{e^{-i k h}}{h + d + 2n d} T_{i0}^n \left( R_{10}^{(n-1)} \right)^2 e^{-i k i d (1+2n)} T_{10}. \quad \text{(A22)}
\]
\[
u_{i n} = -p_{i n} \left( 1 + \frac{1}{i k_0 (h + d + 2n d)} \right). \quad \text{(A23)}
\]

A2.5. Plane Wave Simplification
A different near field correction is applied for each sound wave that travels one or multiple times through the material. For this reason an exact solution cannot easily be obtained for spherical waves. However for plane waves an exact solution can be found by treating each successive wave as part of an infinite series. In case \( h \) goes to infinity \( x \) can simply be replaced by the normalized characteristic material impedance \( Z_{cn} = \frac{Z}{Z_0} \). For a backplate the impedance \( Z_m \) in front of the material becomes
\[
Z_m = Z_{cn} \frac{1 + e^{-2 i k d}}{1 - e^{-2 i k d}} = -i Z_{cn} \tan^{-1} (k_1 d). \quad \text{(A24)}
\]
Especially the last expression \(-i Z_{cn} \tan^{-1} (k_1 d)\) is often found in literature [17, 18]. Without the backplate the impedance in front of the material is
\[
Z_m = \frac{1 + e^{-2 i k d}}{1} - Z_{cn}, \quad \text{(A25)}
\]
which is consistent to the solutions found in e.g. [17]. Behind the material the impedance would be equal to the characteristic impedance of air \( (\rho_0 c) \) and would have little meaning. The ratio of the measured pressures in front \( p_m \) and behind the sample \( p_{m0} \) does depend on the characteristic impedance and wavenumber of the material,
\[
\frac{p_m}{p_{m0}} = \frac{Z_{cn} + 1}{2 e^{-i k d} Z_{cn} - \frac{1}{2}}. \quad \text{(A26)}
\]

References


