In-situ measurement method of ensemble averaged impedance and absorption characteristics of materials at field incidence using p-v or p-p sensors

Toru Otsuru\textsuperscript{a}, Reiji Tomiku
Faculty of Engineering, Oita University, 700 Dannoharu, Oita 870-1192, Japan

Noriko Okamoto
Venture Business Laboratory, Oita University, 700 Dannoharu, Oita 870-1192, Japan

Masahiko Murakami, Fuminiri Kutsukake, Nazli Che Din
Graduate School of Engineering, Oita University, 700 Dannoharu, Oita 870-1192, Japan

ABSTRACT
For practical usages of wave-based computer simulation techniques like FEM and BEM, impedance data of various materials are indispensable. While, there are not enough databases nor measurement techniques that can provide such data as with appropriate accuracy and broad range of adaptability especially when one is applied to room or environmental acoustics. To meet the demand, the authors proposed an impedance measuring method using two microphones, i.e. p-p sensors, and environmental noise (Ref: Y. Takahashi et al., Applied Acoustics 66(7), 2005). Recently, with the help of p-v sensors developed by De Bree, both measurement system and theoretical model of the method were simplified, and ensemble averaged impedance at field incidence can be obtained by the method. Hereafter, the outline of newer method including its repeatability and universality is given in brief, and the results obtained by using p-p and p-v sensors are compared. The results are also compared with those obtained by conventional measurement techniques like tube-method and reverberation method, and some discrepancies among the techniques are confirmed. To prove the method’s superiority as an in-situ impedance measurement technique, BEM investigation is conducted and the results shows that the total measurement system can eliminate diffraction effect caused by specific relation between source and specimen that cannot easily be eliminated by the conventional techniques.

1 INTRODUCTION
With the rapid progress of computer technology, numerical simulations based on wave equations have come to be powerful tools for acoustical investigations and design processes\cite{2}. Although the boundary conditions of such simulations are generally modeled using surface impedance, insufficient amounts of impedance databases might have been supplied to date. What is contradictory is that in many room and environmental acoustic problems within the lower frequency region, roughly less than 1000 [Hz], wave-based considerations would be required, whereas various \textit{in-situ} impedance measurements frequently encounter difficulties because of the wave characteristics of the sound.

To overcome the situation, the authors proposed a method that uses a two-microphone technique and environmental anonymous noise, namely the "EA-noise method"\cite{1}. In the origi-
Figure 1: System geometry and coordinates for a sound source $P_s$ incident on a specimen $F$ with the area of $L_1 \times L_2$ bounded by an infinitely flat hard plane $F_0$.

The general geometry of the system under consideration is depicted in Fig. 1. Here, $\Omega$, $(x, y, z)$, and $(r, \theta, \phi)$ respectively denote the upper half space, Cartesian, and polar coordinate systems and a point source $P_s$ and a specimen $F$ with the area of $L_1 \times L_2$ are assumed to be as portrayed in the figure. An infinite hard-plane $F_0$ is also assumed to surround the specimen; in case the specimen has a thickness $T$, it might be a plane like $z = -T$, $z = 0$, etc.

In the EA-Noise method as is illustrated in Fig. 2, two microphones are placed close to the specimen’s surface, and an impedance value is obtainable with sufficient reproducibility with no loudspeaker if the ambient noise is sufficient [1]. Supplemental noise generated from loudspeaker(s) might be added and a similar measurement can be conducted (pEA-Noise method). Approximated surface normal impedance by the EA-Noise method is given by...
\[
Z_{EA} = \rho_c \frac{H_{ab}(1 - e^{2jk(l+d)}) - e^{jkl}(1 - e^{2jkd})}{H_{ab}(1 + e^{2jk(l+d)}) - e^{jkl}(1 + e^{2jkd})}.
\] (1)

Here, \(c, k, j\) and \(\rho\) sound speed, wave number, \(\sqrt{-1}\) and air density respectively, and \(H_{ab}\) is the transfer function between sound pressures measured by \(M_a\) and \(M_b\).

On the other hand, if a p-v sensor is located at the material’s surface, and let \(p_{surf}\) and \(u_{n, surf}\) be sound pressure and particle velocity normal to the specimen’s surface respectively, then the surface normal impedance is simply defined as Eq. (2):

\[
Z_n(x, y) = \frac{p_{surf}(x, y)}{u_{n, surf}(x, y)}.
\] (2)

In rather practical situations, there might be some distance, say \(d'\), between the material surface and the p-v sensor because of its physical size: 1/2 inch diameter. Detailed discussion of the difference attributable to \(d'\) is given in the following sections. First, let us regard \(d'\) as sufficiently small. Then the following approximation stands.

\[
Z_n \approx \frac{\tilde{p}}{\tilde{u}_n}.
\] (3)

Hereinafter, \(\tilde{p}\) and \(\tilde{u}_n\) denote sound pressure and particle velocity normal to the surface respectively measured using the p-v sensor; the expression \((x, y)\) is omitted.

### 2.1 Ensemble averaged surface normal impedance at random incidence

We next introduce an impedance \(< Z_n >\) that is ensemble averaged impedance over such a sufficient number of incoherent sound source \(P_S(r, \theta, \phi)\) as to expect random incidence, and the impedance can be expressed as Eq. (4a) or (4b),

\[
< Z_n > = \frac{< p_{surf} >}{< u_{n, surf} >},
\] (4a)

\[
< Z_n > = < \frac{p_{surf}}{u_{n, surf}} >,
\] (4b)

where \(< \cdot \>\) denotes ensemble average. The two equations become identical if the system is ergodic and the statistical randomness both in time and space is achieved at every instance. In practical cases, especially in measurements, however, achieving randomness requires time-windowing with some sufficient length as well as some amount of averaging numbers. Therefore, it might be safer to adopt the latter for the definition of \(< Z_n >\) at this stage.
The sensor is fixed at a point \((x, y)\) close to the surface, and averaging is not performed over receiving points (i.e., area averaging), but one can expect to have Eq. (5) because of the randomness of incident sound,

\[
< \theta_t > = 0. \tag{5}
\]

Similar to the oblique incidence case described in the previous section, the Eq. (6) stands.

\[
< Z_n > = \frac{< p_t + p_r >}{< u_t \cos \theta_t - u_r \cos \theta_r >} = \frac{< p_t >}{< u_t \cos \theta_t >}. \tag{6}
\]

The averaging is assumed to be sufficient and Eq. (5) stands. Therefore, one can approximate the right-hand-side of Eq. (6) as

\[
\frac{< p_t >}{< u_t \cos \theta_t >} = \frac{< p_t >}{< u_{n,t} >}, \tag{7}
\]

where \(< u_{n,t} >\) denotes the ensemble average of normal components of transmitted particle velocities.

Next, by equating the right-hand-side of Eq. (7) and \(< Z_2 >\), we can obtain

\[
< Z_n > = < Z_2 >. \tag{8}
\]

Therefore, \(< Z_n >\) can be regarded as an expected value of \(Z_2\), which is derived from Eq. (4b) without assuming "local reactivity" throughout the procedure.

Consequently, by comparing Eqs. (3) and (4b), the following equation is expected to yield a statistically good approximation of the surface normal impedance of a specimen in a practical measurement,

\[
< Z_n > \approx \frac{< \tilde{p} >}{< \tilde{u}_n >}. \tag{9}
\]

The corresponding absorption coefficient \(< \alpha >\) is calculated as

\[
< \alpha > = 1 - \left( \frac{< Z_n > - Z_1}{< Z_n > + Z_1} \right)^2. \tag{10}
\]

3 COMPARISON OF ABSORPTION COEFFICIENTS MEASURED BY DIFFERENT METHODS

In the authors’ previous paper, basic agreement of the absorption characteristics obtained by p-p and p-v sensors were confirmed [3]. Herein, the absorption coefficients of two materials obtained by EA-Noise method with p-v sensor are compared with those obtained by the tube method (ISO 10534-2) and by the reverberation method (JIS A 1409, ISO 354).

The specimens to be measured are a glass wool with 32 [kg/m\(^3\)] and 50 [mm] thick, and Needle felt with 10 [mm] thick. In both EA-noise and reverberation methods, the same samples backed by an 15 [mm] thick acrylic resin plate were measured in a reverberation room with non-parallel walls and with 165 [m\(^3\)].

The specimens’ sizes are 0.9 [m] \(\times\) 1.8 [m] for EA-Noise method, while 2.7 [m] \(\times\) 3.0 [m] for reverberation method. For tube method, small circular samples with the radius of 10 [cm] cut off from the same materials’ lots are measure.

Seven speakers including a sub-woofer that radiate pink noise were employed as sound sources in the EA-Noise method, or pEA-Noise method, and p-v sensor was set at 1[cm] above the center of a sample.
In Fig. 3, absorption coefficients obtained by the three methods are compared. On the whole, the agreement of the three methods itself is not very good at this stage, but the tendencies according to the materials’ characteristics can be grasped by the EA-Noise method.

The results for the glass wool obtained by the reverberation method show the highest absorption coefficients among the three methods, which can be regarded as to be caused by the area effect. The absorption coefficients for the needle felt obtained by the EA-Noise method showed fair agreement with that of the tube method, on the other hand, there exists some discrepancy between the results of the two methods for the glass wool. The reasons are investigated by using the numerical simulation in the following section.

4 MATH-PHYSICAL MODEL AND NUMERICAL CONFIRMATION USING BOUNDARY ELEMENT METHOD

Kawai proposed and conducted a numerical simulation using a boundary integral equation to investigate the area effect of sound-absorbent surfaces at the random incidence condition.

We follow Kawai’s method and simulate the phenomena by which ensemble averaged impedance is to be measured using the procedure described above to clarify whether the ensemble averaging makes any sense. Unlike Kawai’s procedure, we do not calculate any energy; rather we calculate a series of sound pressure $p_{surf}$ and particle velocity $u_{n,surf}$ using commercial software of BEM (WAON ver.2.0; Cybernet Inc.), then corresponding $< Z_n >$ can be reduced using Eq. (4b).

4.0.1 Normal incidence simulation

Initially, rather simple normal incidence ($\theta = 0$) cases for three isotropic materials are simulated and resulting $Z_{n,0}$ and corresponding $\alpha_0$ are compared to those derived from Miki’s empirical equations for an infinite porous material ($Z_{emp}$ and $\alpha_{emp}$ respectively). To realize the plane wave incidence, a point source $P_3$ is located at (0, 0, 500), or $r = 500$ [m] distance.

Three sizes of glass wool, with $L (= L_1 = L_2) = 1, 2$ and 4 [m], all have the same 32 [kg/m$^3$] density and 10 000 [N s/m$^4$] flow resistivity. They are simulated as specimen $F$. The thickness $T$ is set to zero in the BEM’s geometrical configurations. However, simultaneously, $T = 0.05$ [m] is assumed for all three glass wool specimens to determine $Z_{emp}$. The determined $Z_{emp}$ is assigned as the impedance condition for $F$ in the BEM simulation to be conducted.

Then, the surface impedance $Z_{n,0}$ at the center of the specimen is calculated using (2). Here, to check the effect of $d'$, the distance $d'$ is set to 1 [cm], and both pressure and particle velocity
at the point (0, 0, 0.01) are calculated in the frequency domains of 25–500 [Hz] at a 25 [Hz] interval, and 500–1000 [Hz] at a 50 [Hz] interval.

The results are presented in Fig. 4 and the intermediate lines of both simulated $Z_{n,0}$ and corresponding $\alpha_0$ agree well with $Z_{\text{emp}}$ and $\alpha_{\text{emp}}$ respectively, provided the wavelike fluctuations are omitted. It is noteworthy that the absorption coefficient of a specimen with a finite size can be approximated using the method to fit that of the same material with infinite size. In addition, the effect of $d'$ can be regarded as sufficiently small in these cases.

Nonetheless, discrepancies exist, i.e. wavelike fluctuations, in the frequency region of 100–800 [Hz]. The discrepancies can be regarded as resulting from interferences between direct wave and reflected waves from the edges around the specimen. In-phase and out-of-phase frequencies that cause the interferences, $f_{\text{in}}$ and $f_{\text{out}}$ respectively, can be estimated easily using Eqs. (11) and (12) on the condition of $r >> L$ and $\lambda$, i.e. wave length.

$$f_{\text{in}} = \frac{c}{\Delta L} \times n,$$

(11)

$$f_{\text{out}} = \frac{c}{\Delta L} \times \frac{2n - 1}{2},$$

(12)

where $\Delta L = L/2$, which is the path length difference between the direct sound wave and a representative reflected sound wave, and $n = 1, 2, \cdots$. Because the specimens have rectangular shapes, the $\Delta L$ lies between $L/2$ and $\sqrt{2}L/2$. The frequencies for the case of both $\Delta L = (L/2 + \sqrt{2}L/2)/2$ and $c = 340$ [m/s] are calculated and plotted into Fig. 4(c).

The frequencies around which the wavelike fluctuations are portrayed in Fig. 4(a) and 4(b) coincide well with the in-phase and out-of-phase frequencies. Therefore, the issue can be reduced to sound absorption characteristics might be measured using such a method using a sound source, but the wavelike fluctuations resulting from interference might be unavoidable.

The larger the specimen’s size, the more numerous but smaller in amplitude the wavelike fluctuation becomes. However, because it is usual for buildings to use such a material as with $L \approx 1$ [m], an in-situ absorption measurement of the material using some reflection method shall be interfered.

Figure 4(b) also shows that, in the case where the material has $L = 4$ [m] size, good results without distinct fluctuation can be expected only greater than 700 [Hz], whereas wavelike distortions are unavoidable below that frequency.

### 4.0.2 Random incidence simulation

Next, the same BEM calculations except for the sound source condition are carried out for the same glass wool ($L = 1$ [m]). Point sources are located at equally distributed $m = 206$ points on the sphere with the radius $r = 500$ [m]. The sound source number is slightly less than that of Kawai’s model, but we conducted several preliminary simulations and confirmed that the difference is negligible.

A series of BEM calculations are conducted changing the sound source simultaneously; the phases and amplitudes of the sound sources are randomized to average. A series of random numbers that follow Gaussian distribution $N(\mu, \sigma^2)$ with mean $\mu$ and variance $\sigma^2$ are generated for the modeling of sound source amplitude. Similarly, a series of random numbers that follow the unique distribution $[-\pi, \pi]$ for the phase. The values of $\mu$ and $\sigma$ that correspond to 120 [dB] and 20 [dB] are assigned respectively, and with a set of calculated $p_{\text{surf}}$ and $u_{n,\text{surf}}$, the ensemble averaged impedances are calculated using Eq. (4b).
As presented in Fig. 5, the wavelike fluctuations that are found in the normal incident simulations (Fig. 4(a) and 4(b)) can be flattened and the simulated $< Z_{n} >$ and corresponding $< \alpha >$ agree fairly well, respectively, with assigned $Z_{emp}$ and corresponding $\alpha_{emp}$. The result alleviates the second issue that the ensemble averaged surface normal impedance gives an excellent approximation of surface normal impedance of the material with no distinct fluctuation caused by the interference. Even if the specimen’s size is $L = 1$ [m], the equation $< Z_{n} > \simeq Z_{emp}$ stands.

5 EXPERIMENTAL CONFIRMATION

5.1 Measurement outline and comparison with simulation

Two types of measurement are conducted to support the issues inferred from results of the BEM simulations: one in an anechoic room with the volume of 58 [m$^3$], the other in the reverberation room utilized in the former sections. Both rooms are at the Computing Center of Oita University. The measurements follow the basic procedure described in the previous paper[1] except that the sensor is not an array of microphones; instead, it is a p-v sensor.

Because the environmental noises inside are insufficient in such rooms, an additional speaker(s)
Figure 5: Comparisons of absorption characteristics of the glass wool, simulated \((L=1 \text{ [m]}, \text{ random incidence, ensemble averaged})\) vs. empirical (infinite, normal incidence): (a) normalized surface normal impedances \(\langle Z_n \rangle / \rho c\) and \(Z_{\text{emp}} / \rho c\); (b) corresponding absorption coefficients \(\langle \alpha \rangle\) and \(\alpha_{\text{emp}}\).

is (are) used as follows: in the anechoic room, a loudspeaker (10 cm single-cone speaker in a box), is located at \(r = 2 \text{ [m]}\) distance normal to the glass wool to realize the normal incidence condition. In the reverberation room, seven loudspeakers including a sub-woofer generating incoherent pink noise are distributed over the floor to realize the random incidence condition.

In each room, a glass wool with the sizes of \(0.9 \text{ [m]} \times 0.9 \text{ [m]} \times 0.05 \text{ [m]}\), and the density of \(32 \text{ [kg/m}^3\) is laid on the floor. The four sides of the glass wool are covered with thin aluminum plates. In the anechoic room the glass wool is laid on a \(0.02 \text{ [m]}\) wooden plate, whereas in the reverberation room it is laid on the concrete floor directly. The glass wool size \((L = 0.9 \text{ [m]})\) is not exactly identical to that of the simulated one in the previous section \((L = 1 \text{ [m]}\)), but we expect to have sufficient validity for fundamental discussion at this stage.

Then, a p-v sensor (PT0406-7; Microflown) is placed at the center of the glass wool with \(d' = 1 \text{ [cm]}\), and the sensor’s outputs are plugged into a 2ch Fast-Fourier-Transform instrument (FFT, SA-78; Rion Co. Ltd.). The FFT resolution is set to \(1.0 \text{ [Hz]}\) and a Hamming window is used. Measured data are averaged 30 times.

5.1.1 Comparison at normal incidence

In Fig 6, the normalized surface normal impedances and corresponding absorption coefficients of the glass wool at normal incidence condition obtained using the measurement are compared to those obtained by simulation. The measurement is conducted in the anechoic room and the simulated values are those of \(L = 1 \text{ [m]}\) case shown in Figs. 4(a) and 4(b). Again, the wavelike fluctuations are readily apparent in the measured data, but the basic tendency of measured data
Figure 6: Impedances and absorbent coefficients of glass wool at normal incidence. Comparisons of measured ($L = 0.9 \, [\text{m}]$), simulated ($L = 1 \, [\text{m}]$) and empirical (infinite): (a) normalized surface normal impedances, $Z_{n0}/\rho c$ ($Z_{\text{meas}}$ vs. $Z_{\text{sim}}$ and $Z_{\text{emp}}$); (b) corresponding absorption coefficients, $\alpha_0$ ($\alpha_{\text{meas}}$ vs. $\alpha_{\text{sim}}$ and $\alpha_{\text{emp}}$).

agrees well with that of the simulation.

The experimental result also supports the first issue: the wavelike fluctuations are unavoidable for a measuring method with a normal incidence sound source. The measurement is conducted in a rather small anechoic room, and the distance $r$ or the distance from the specimen to the ceiling or side walls is not so far away that there might be reflections or some more interference effects to disturb the measurement. Additionally, in the lower frequency region of less than $200 \, [\text{Hz}]$, many distortions are visible in the measured values, both $Z_n$ and $\alpha$, which suggests that the sound energy in the region might be insufficient.

5.1.2 Comparison at random incidence

The sound absorption characteristics of the glass wool are measured in the reverberation room for comparison to the BEM simulation using ensemble averaging. In Fig. 7(a) measured and simulated normalized surface normal impedances $< Z_n >$ at random incidence conditions are compared. The Fig. 8(b) is a comparison of the corresponding absorption coefficients $< \alpha >$. No distinct fluctuation is observed in measured $< Z_n >$ nor measured $< \alpha >$, which supports the second issue that the ensemble averaging is sufficiently effective to reduce the wavelike fluctuations in resulting absorption characteristics.

However, a non-negligible discrepancy remains between measured and simulated data in the frequency region of $100$–$900 \, [\text{Hz}]$. From comparison of the results between normal and random incidence conditions, we considered that the discrepancy results from anisotropy of the glass
5.2 Improved simulation with anisotropy consideration

5.2.1 Consideration of anisotropy of glass wool into BEM simulation

For glass wool, angular-dependent normal impedance $Z_{n,\theta}$ can be estimated using Eq. (13) which was given by Allard[15], and originated by Pyett[16]; knowing the planar and normal wavenumbers $k_p$ and $k_n$ in the glass wool,

$$Z_{n,\theta} = (k_n Z_n/k_q) \coth(\gamma_q T). \tag{13}$$

The quantities $k_q$ and $\gamma_q$ are equal to

$$k_q^2 = k_n^2 \{1 - (k_0/k_p)^2 \sin^2 \theta\}, \tag{14}$$

$$\gamma_q = i k_q, \tag{15}$$

where $k_0$ is the wavenumber in air.

A series of procedures given by Allard[17] is carried out to determine the $k_p$ and $k_n$. First, several preliminary measurements using an acoustic tube were carried out and a normal flow resistivity, $\sigma_n$, of 10 000 [N s/m$^4$] was chosen for the glass wool to fit the predicted normal surface impedances given by the empirical equations[14][18]. Next, the anisotropy parameter $s = \sigma_p/\sigma_n$ was inferred to be equal to 0.6, in which $\sigma_p$ denotes the planar flow resistivity.
Figure 8: Improvement of simulation by considering anisotropy of glass wool. (a) normalized surface normal impedances \( \frac{Z_n}{\rho c} \) \( Z_{sim\_with} \) vs. \( Z_{sim\_without} \) and \( Z_{meas} \); (b) corresponding absorption coefficients \( \alpha \) \( \alpha_{sim\_with} \) vs. \( \alpha_{sim\_without} \) and \( \alpha_{meas} \).

Finally, \( k_p \) and \( k_n \) are calculated as propagation constants using empirical equations[17][19], and \( Z_{n,\theta} \) is obtainable.

5.2.2 Comparison with measurement

Into each BEM simulation of an incidence angle \( \theta \), the \( Z_{n,\theta} \) obtained by Eq. (13) is given as the boundary condition for \( F \) instead of \( Z_n \), and by summing up the resulting \( p_{surf} \) and \( u_{n,\surf} \), the ensemble averaged impedance taking into account of anisotropy of the glass wool, and corresponding absorption coefficient are calculated.

The results are presented in Fig. 8(a) and 8(b) together with data both of the original simulation and of measurement that are portrayed in Fig. 7 for comparison. Remarkable refinements in the agreements between simulated and measured values can be achieved and the agreement becomes good.

Therefore, the issue described above is confirmed: ensemble averaged surface normal impedance can provide an appropriate expected value of specific acoustic impedance of a material. In the ensemble averaging process, the incident-angle-dependent characteristics of the medium, if any, are properly incorporated.

6 CONCLUSIONS

The pertinent concept of ensemble averaged surface normal impedance to be measured using an in-situ technique and its math-physical model have been presented. Several BEM simulations of glass wool both at normal and at random incidences show that ensemble averaging decreases the
interference effect caused mainly by the specimen’s edges and that the ensemble averaged surface normal impedance at random incidence gives an appropriate expected value of the surface normal impedance of the material. Measurements of two types were conducted for comparison with the simulations, which show plausible agreements to support the issues. Further numerical and experimental investigations are now being pursued intensively.

7 ACKNOWLEDGEMENT
This research is partially supported by a Ministry of Education, Science, Sports and Culture, Grant-in-Aid for Exploratory Research, 19656143, 2007–8, and is conducted as one Research Project (A) at the Venture Business Laboratory in Oita University.

REFERENCES


